Fundamentals of Synthetic Aperture Radar Tomography

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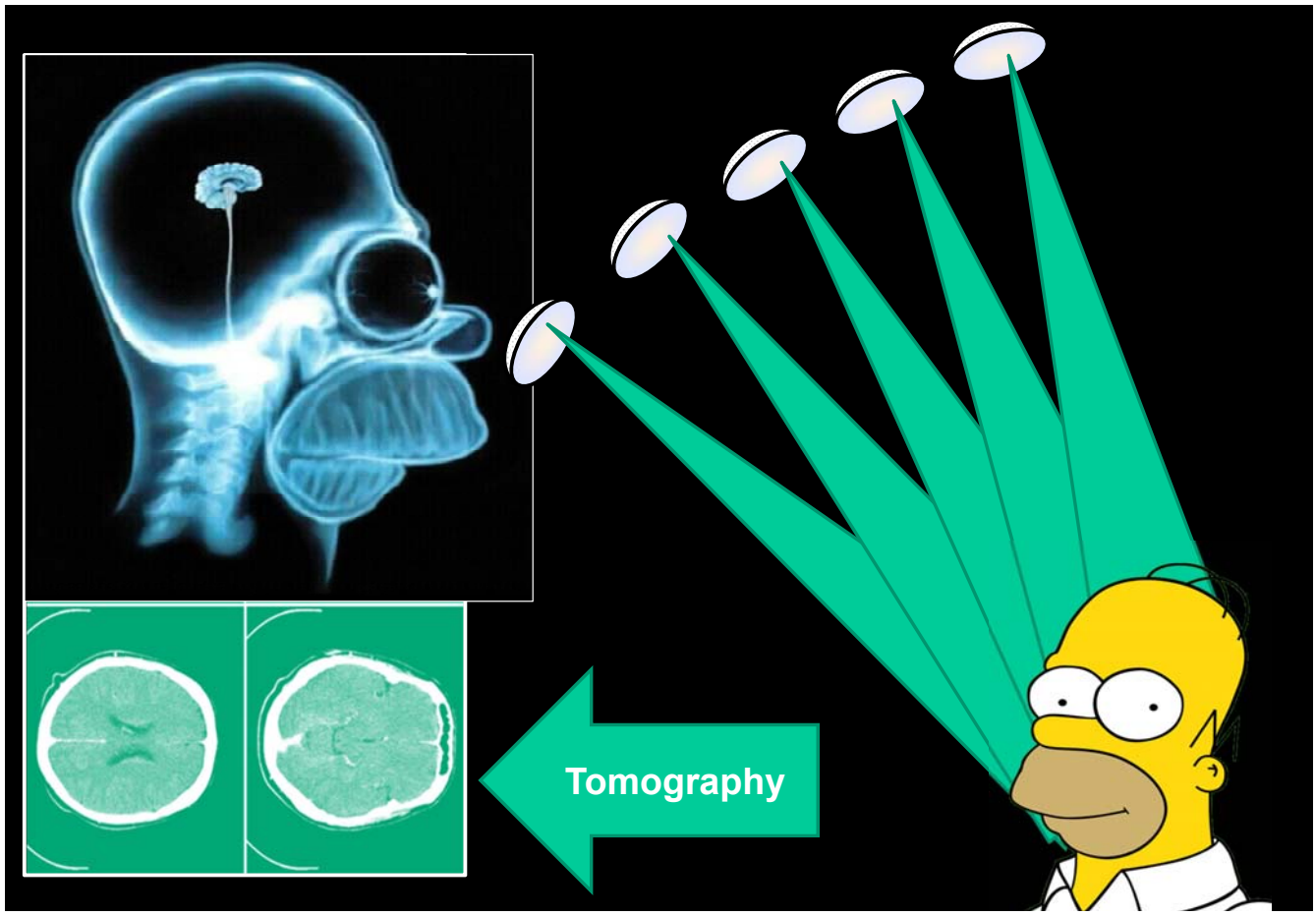
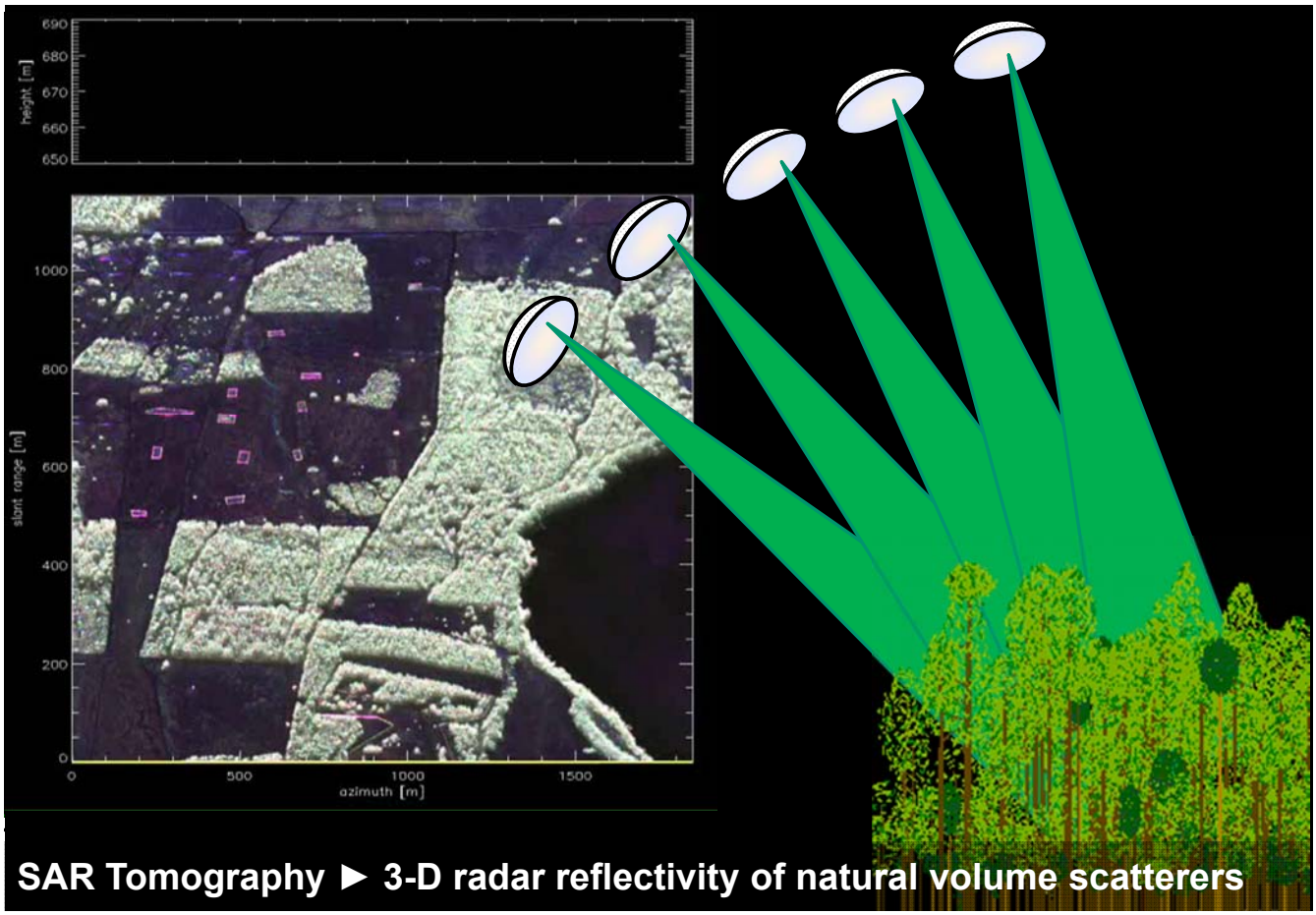
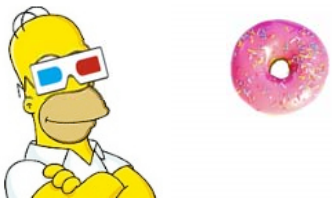




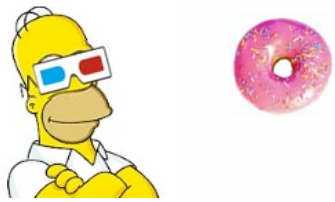
SAR Tomography ► 3-D radar reflectivity of natural volume scatterers



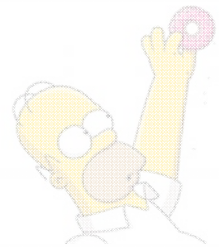
► How to set up an observation space for SAR Tomography

► How to extract 3-D information content from SAR Tomographic acquisitions

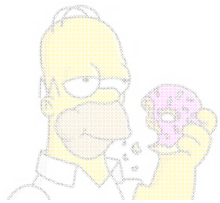
► How to use SAR Tomography in the applications



► **How to set up an observation space for SAR Tomography**

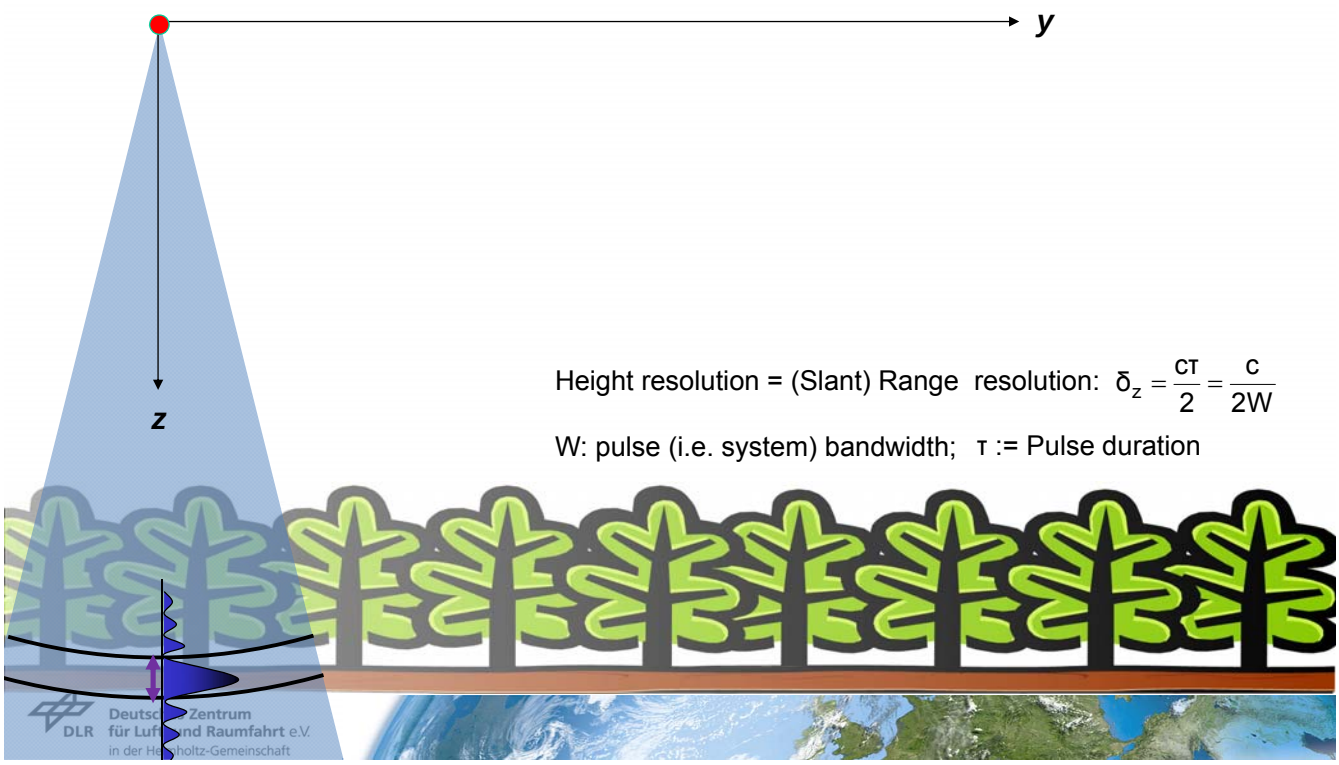


► How to extract 3-D information content from SAR Tomographic acquisitions



► How to use SAR Tomography in the applications

Vertical Resolution: Nadir Looking Case



Height resolution = (Slant) Range resolution: $\delta_z = \frac{c\tau}{2} = \frac{c}{2W}$

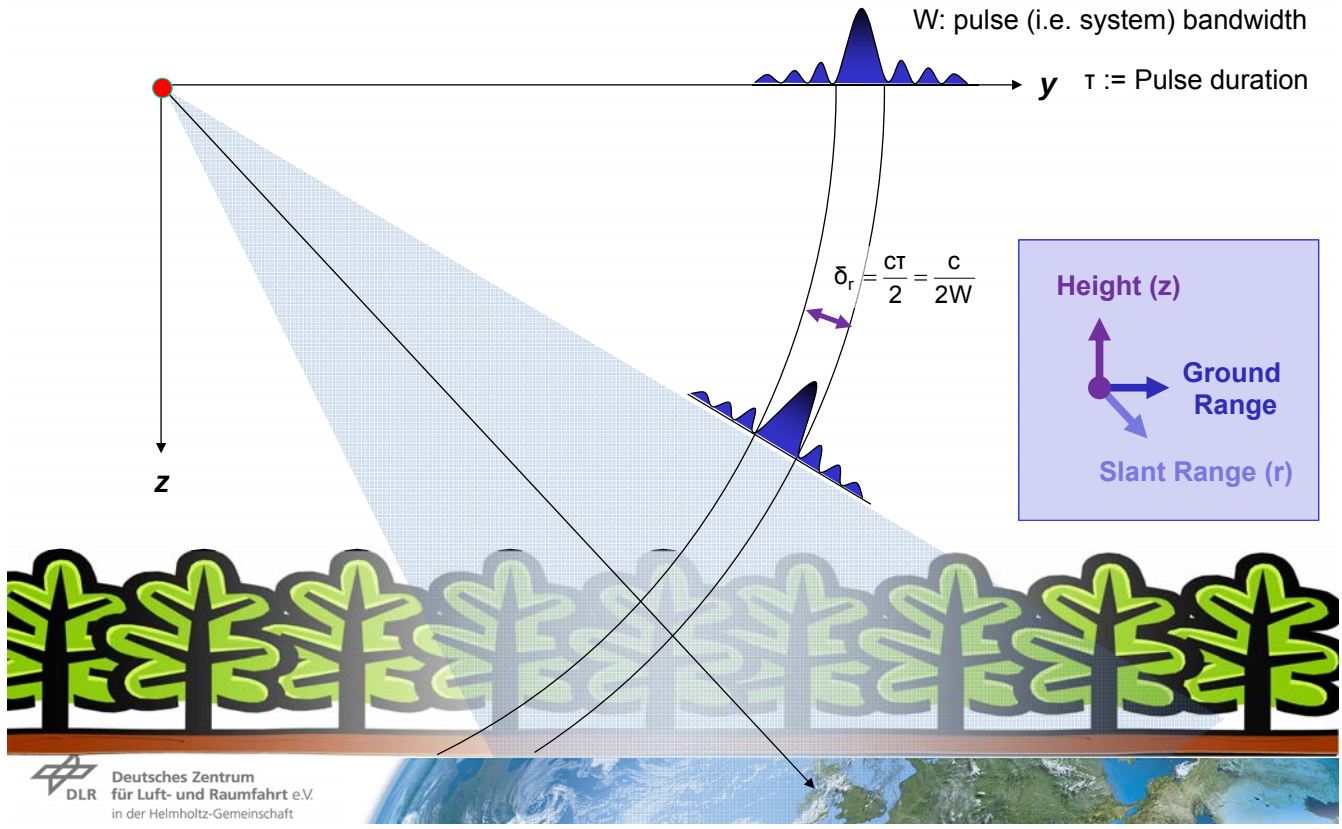
W: pulse (i.e. system) bandwidth; τ := Pulse duration

Vertical Resolution: Side Looking Case

(Slant) range resolution: $\delta_r = \frac{c}{2W}$

W: pulse (i.e. system) bandwidth

τ := Pulse duration



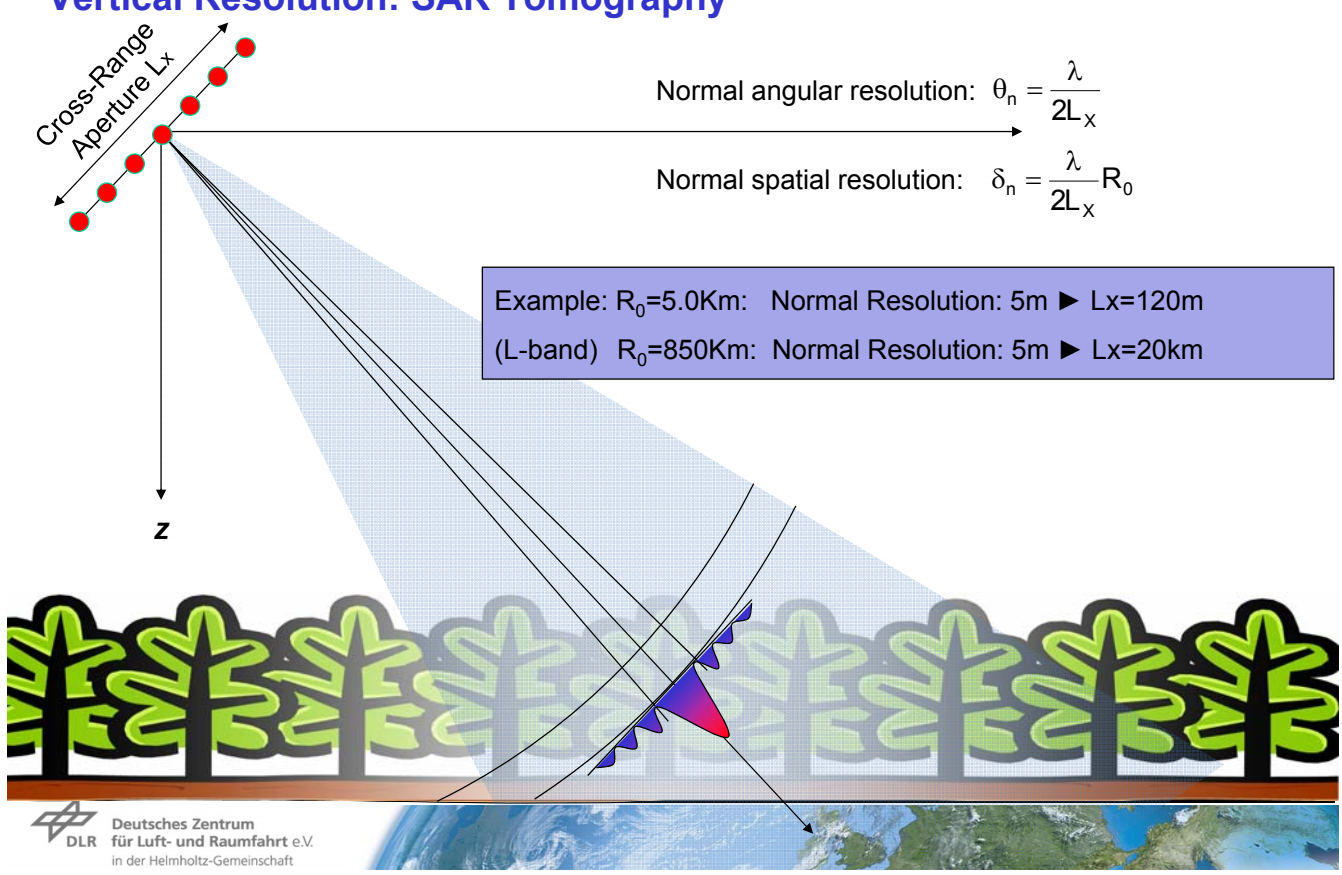
Vertical Resolution: SAR Tomography

Normal angular resolution: $\theta_n = \frac{\lambda}{2L_X}$

Normal spatial resolution: $\delta_n = \frac{\lambda}{2L_X} R_0$

Example: R_0=5.0Km: Normal Resolution: 5m ► Lx=120m

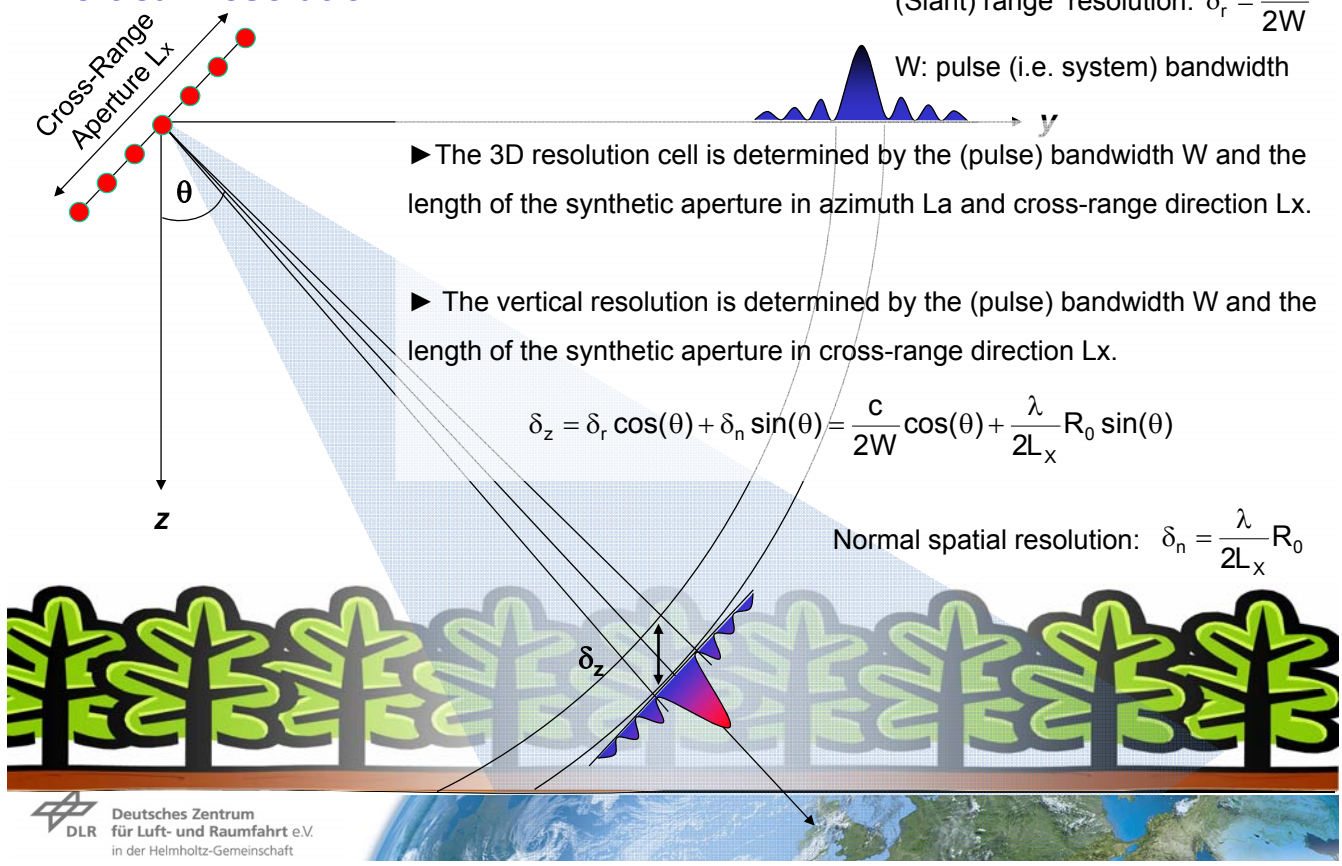
(L-band) R_0=850Km: Normal Resolution: 5m ► Lx=20km



Vertical Resolution

(Slant) range resolution: $\delta_r = \frac{c}{2W}$

W: pulse (i.e. system) bandwidth



►The 3D resolution cell is determined by the (pulse) bandwidth W and the length of the synthetic aperture in azimuth La and cross-range direction Lx.

► The vertical resolution is determined by the (pulse) bandwidth W and the length of the synthetic aperture in cross-range direction Lx.

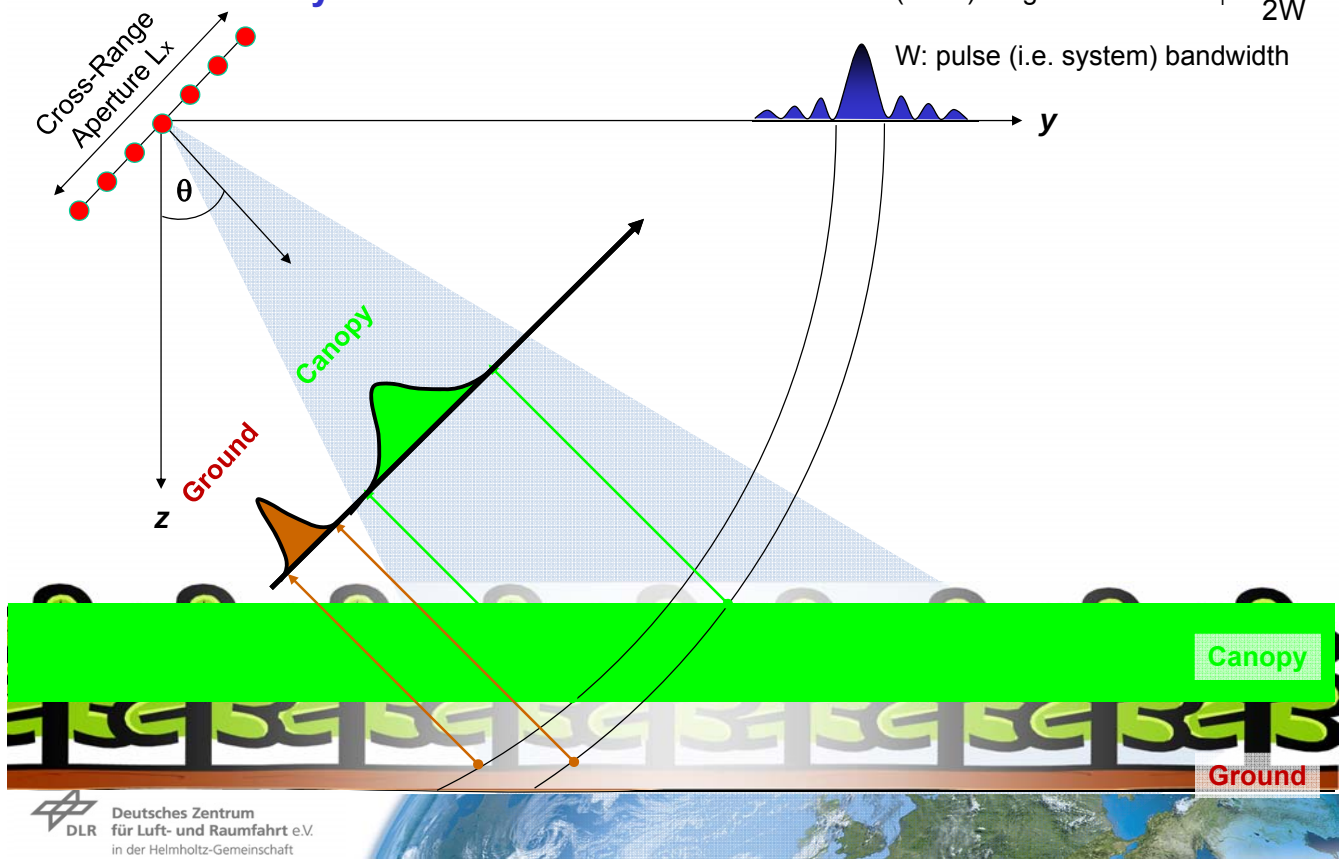
$$\delta_z = \delta_r \cos(\theta) + \delta_n \sin(\theta) = \frac{c}{2W}\cos(\theta) + \frac{\lambda}{2L_X} R_0 \sin(\theta)$$

Normal spatial resolution: $\delta_n = \frac{\lambda}{2L_X} R_0$

The Effect of System Bandwidth

(Slant) range resolution: $\delta_r = \frac{c}{2W}$

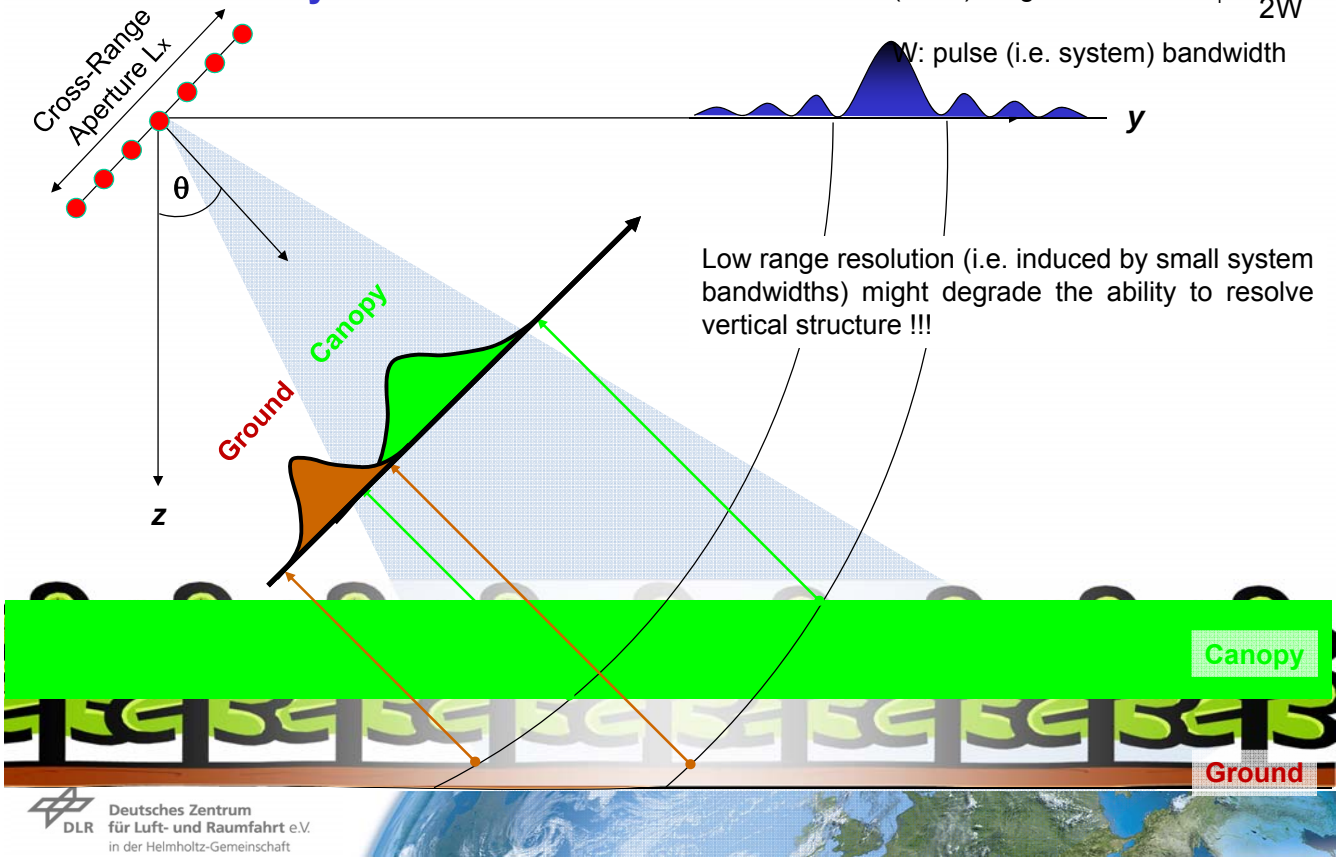
W: pulse (i.e. system) bandwidth



The Effect of System Bandwidth II.

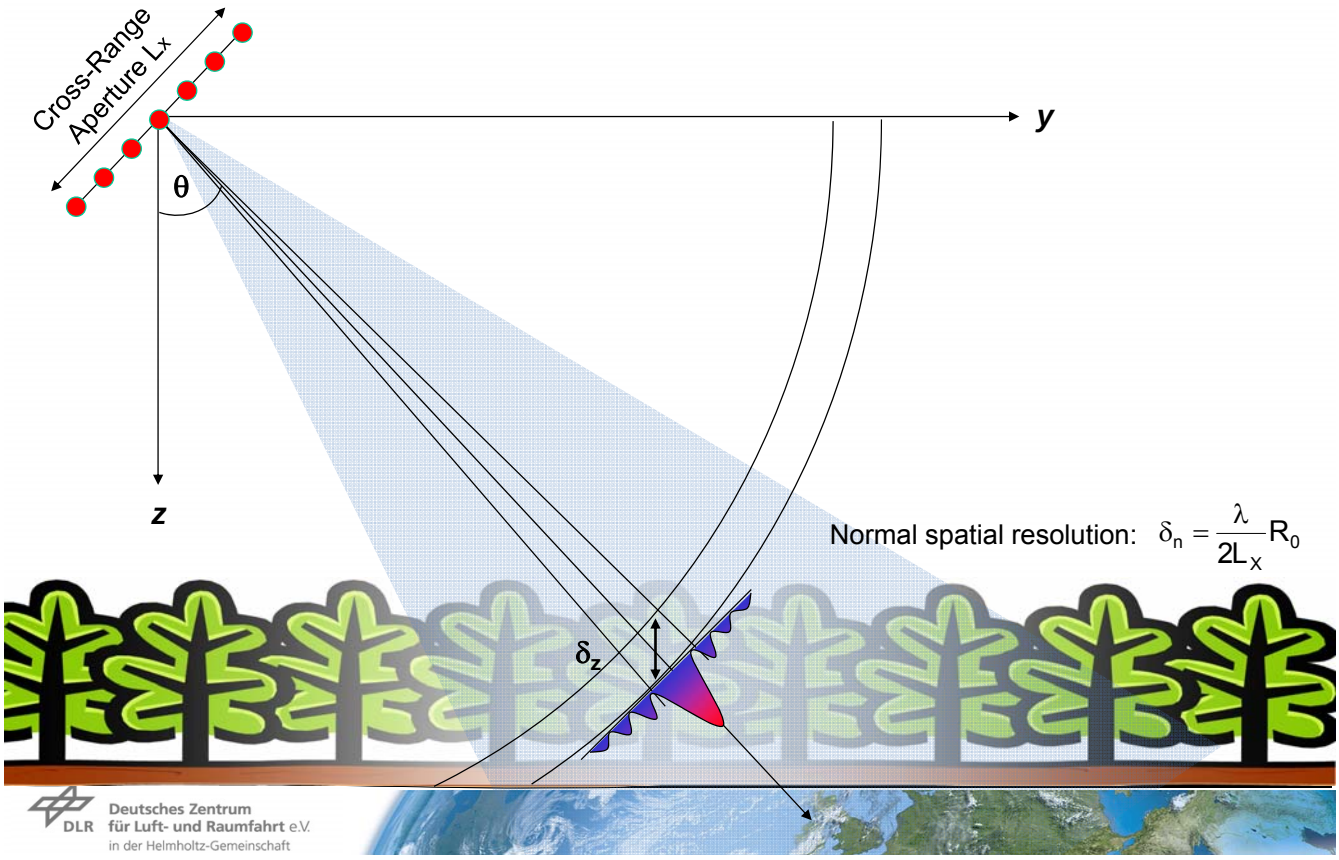
(Slant) range resolution: $\delta_r = \frac{c}{2W}$

W: pulse (i.e. system) bandwidth



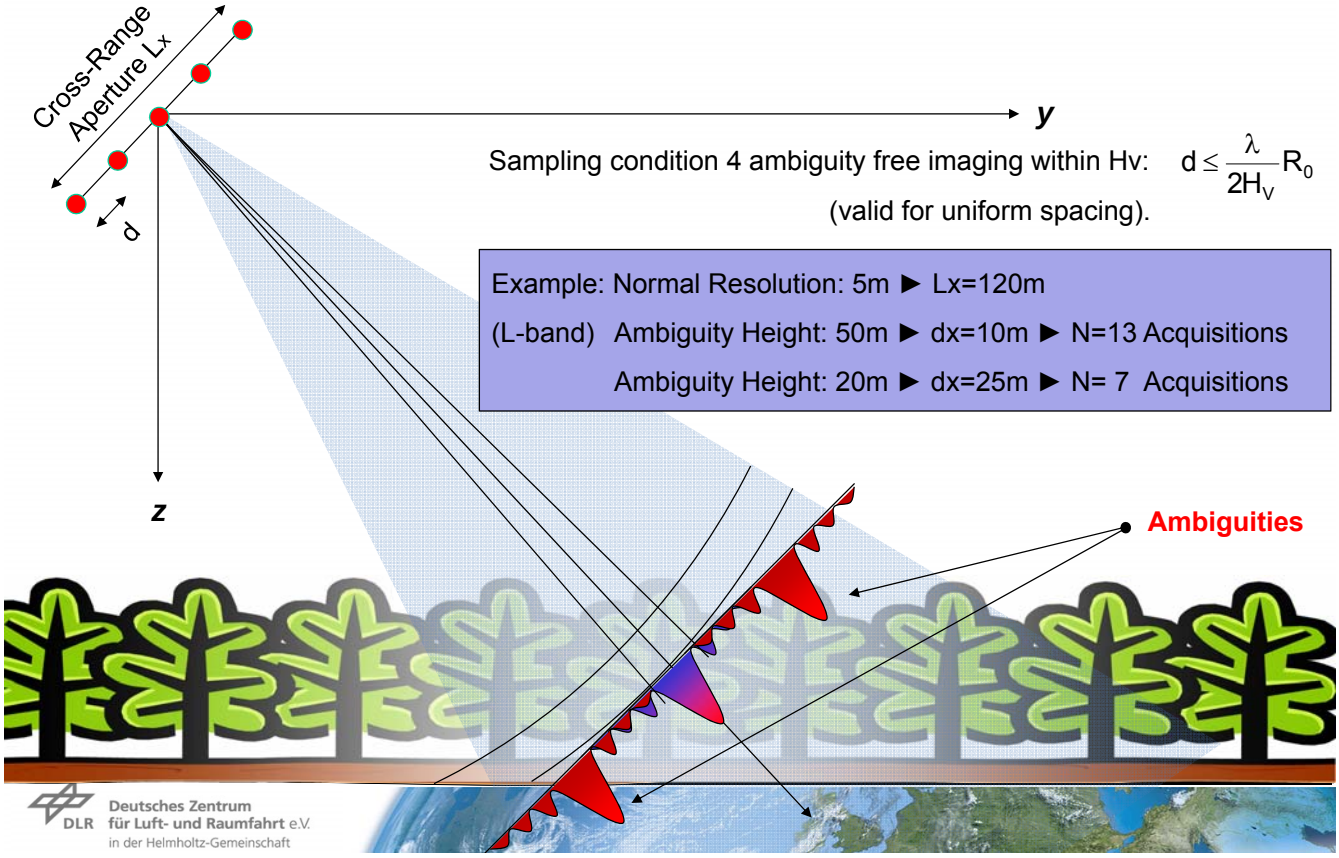
Low range resolution (i.e. induced by small system bandwidths) might degrade the ability to resolve vertical structure !!!

Vertical Resolution



Normal spatial resolution: $\delta_n = \frac{\lambda}{2L_X} R_0$

Vertical Resolution



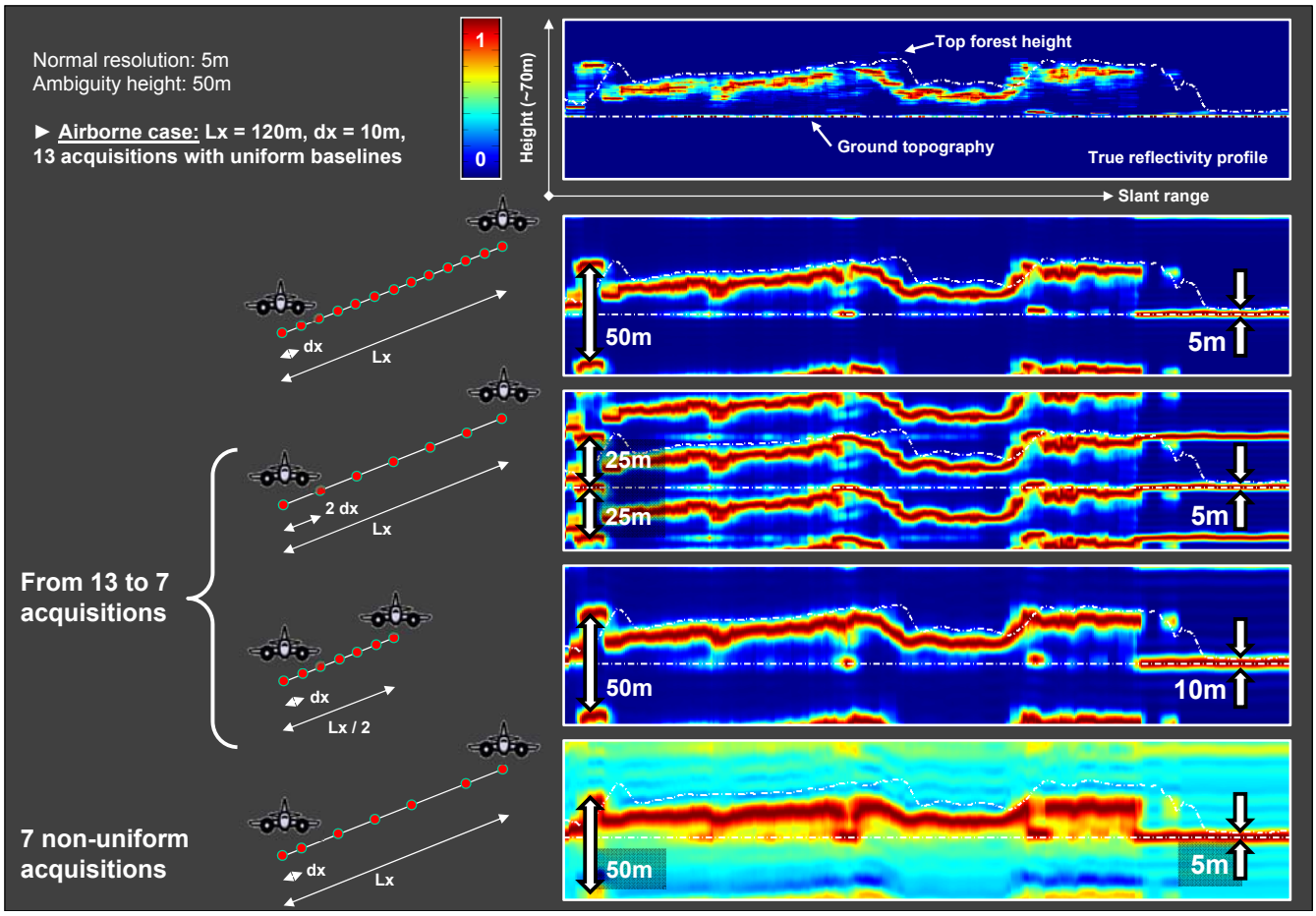
Sampling condition 4 ambiguity free imaging within Hv: $d \leq \frac{\lambda}{2H_V} R_0$

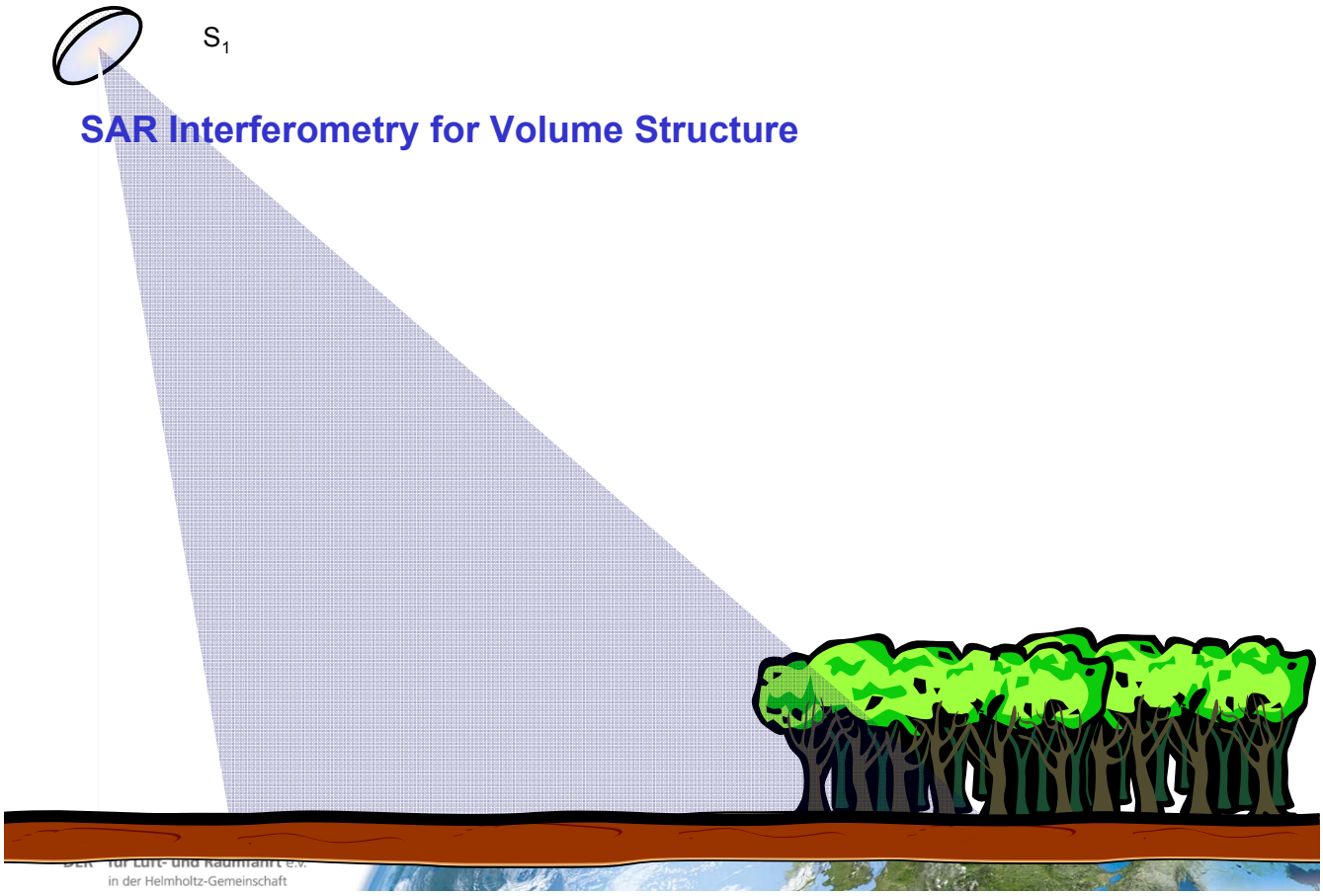
(valid for uniform spacing).

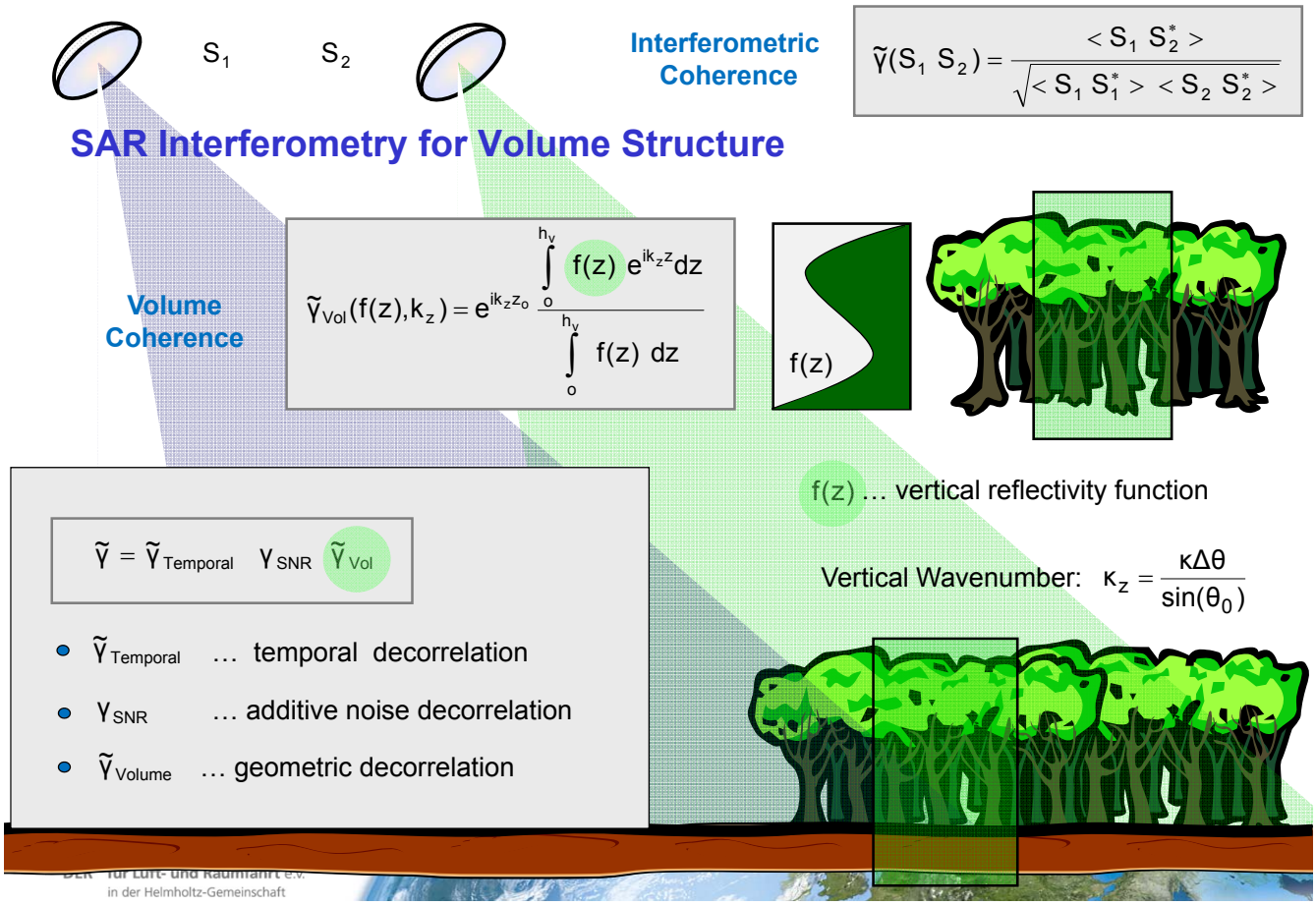
Example: Normal Resolution: 5m ► Lx=120m

(L-band) Ambiguity Height: 50m ► dx=10m ► N=13 Acquisitions

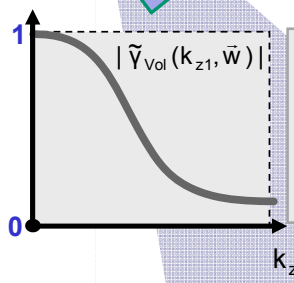
Ambiguity Height: 20m ► dx=25m ► N= 7 Acquisitions



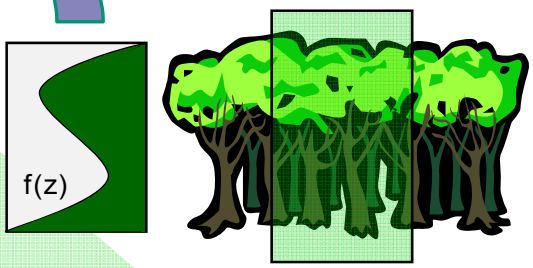

S_1
SAR Interferometry for Volume Structure
in der Helmholtz-Gemeinschaft


S_1
S_2
Interferometric Coherence
$\tilde{\gamma}(S_1\ S_2) = \frac{< S_1\ S_2^* >}{\sqrt{< S_1\ S_1^* > < S_2\ S_2^* >}}$
SAR Interferometry for Volume Structure
Volume Coherence
$\tilde{\gamma}_{Vol}(f(z),k_z) = e^{ik_z z_0} \frac{\int_0^{h_v} f(z)\ e^{ik_z z} dz}{\int_0^{h_v} f(z)\ dz}$
f(z)
f(z) … vertical reflectivity function
Vertical Wavenumber: $\kappa_z = \frac{\kappa \Delta\theta}{\sin(\theta_0)}$
$\tilde{\gamma} = \tilde{\gamma}_{Temporal}\ \gamma_{SNR}\ \tilde{\gamma}_{Vol}$
$\tilde{\gamma}_{Temporal}$ … temporal decorrelation
γ_{SNR} … additive noise decorrelation
$\tilde{\gamma}_{Volume}$ … geometric decorrelation
in der Helmholtz-Gemeinschaft

Normalised Fourier Transform of the vertical reflectivity function f(z)



$$\tilde{\gamma}_{Vol}(k_{z1}) = e^{ik_{z1}z_o} \frac{\int_o^{h_v} f(z)\, e^{ik_{z1}z} dz}{\int_o^{h_v} f(z)\, dz}$$



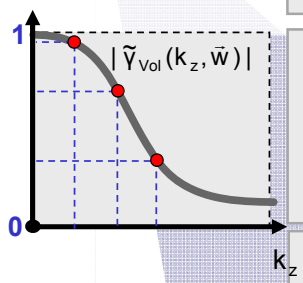
f(z) … vertical reflectivity function

Vertical Wavenumber: $\kappa_z = \frac{\kappa \Delta\theta}{\sin(\theta_0)}$

Multibaseline SAR Interferometry

Baseline 3 (k_{z3})

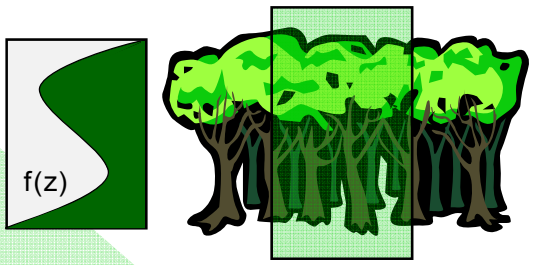
$$\tilde{\gamma}_{Vol}(k_{z3}) = e^{ik_z z_o} \frac{\int_o^{h_v} f(z)\, e^{ik_{z3}z} dz}{\int_o^{h_v} f(z)\, dz}$$



$$\tilde{\gamma}_{Vol}(k_{z1}) = e^{ik_{z1}z_o} \frac{\int_o^{h_v} f(z)\, e^{ik_{z1}z} dz}{\int_o^{h_v} f(z)\, dz}$$

Baseline 2 (k_{z2})

$$\tilde{\gamma}_{Vol}(k_{z2}) = e^{ik_z z_o} \frac{\int_o^{h_v} f(z)\, e^{ik_{z2}z} dz}{\int_o^{h_v} f(z)\, dz}$$

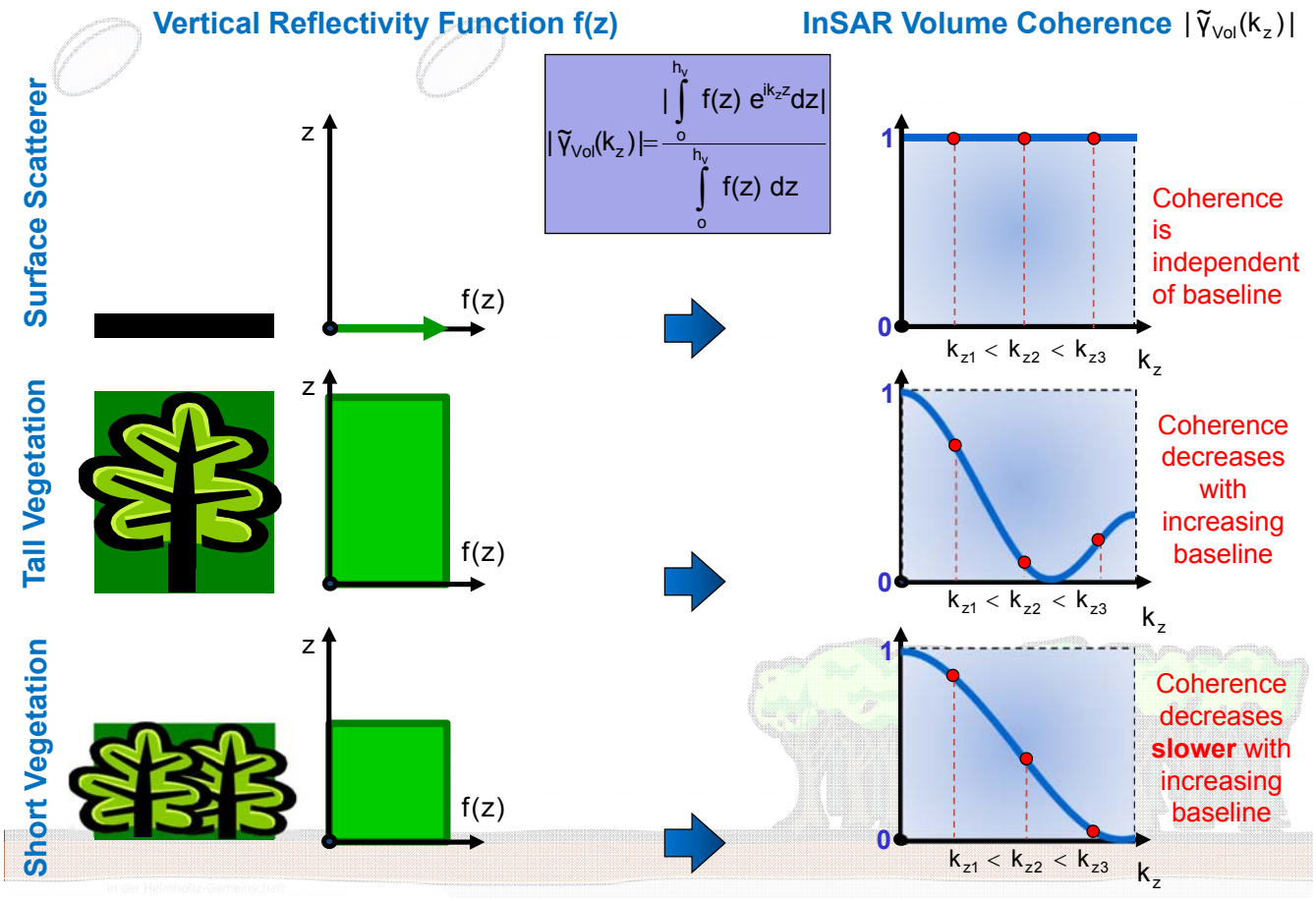


f(z) … vertical reflectivity function

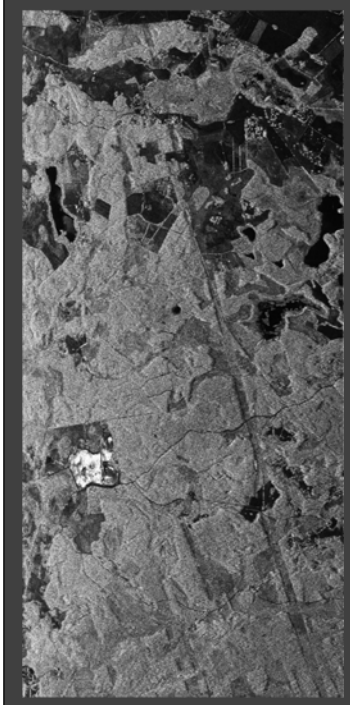
Vertical Wavenumber: $\kappa_z = \frac{\kappa \Delta\theta}{\sin(\theta_0)}$

Multibaseline SAR Interferometry

Multi-baseline measurements allow to sample **the spectrum of the vertical reflectivity** FT{f(z)} **@ different (spatial) frequencies** (k_z).



Amplitude Image

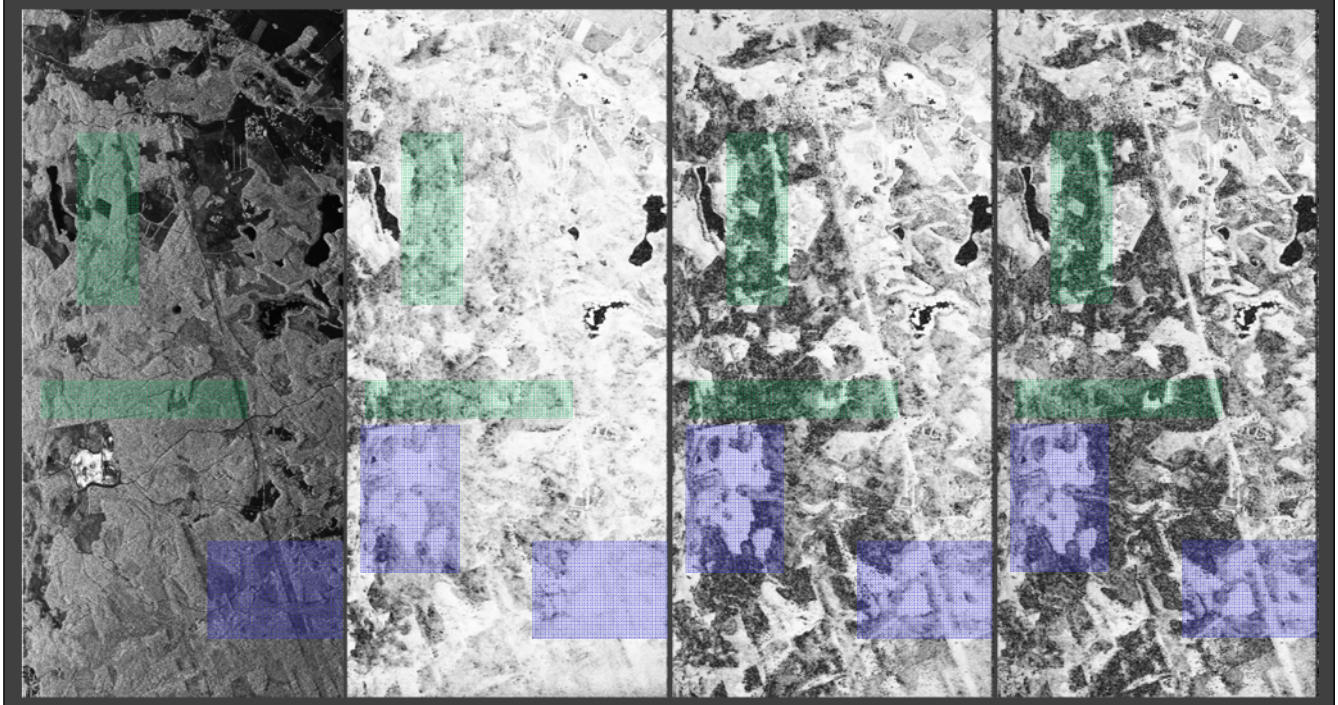


Amplitude Image HH

DLR

0 1

Interferometric Coherence: Volume Decorrelation



DLR

Polarimetric SAR Interferometry

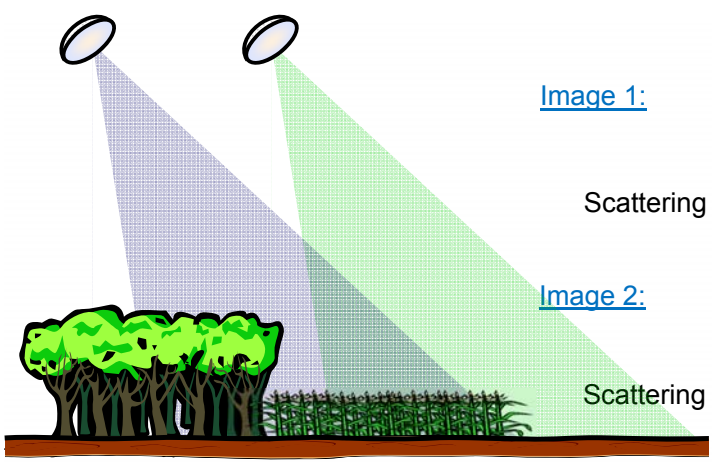


Image 1: Scattering Matrix: $[S_1] = \begin{bmatrix} S^1_{HH} & S^1_{HV} \\ S^1_{VH} & S^1_{VV} \end{bmatrix}$

Scattering Vector 1: $\vec{k}_1 = \frac{1}{\sqrt{2}}[S^1_{HH} + S^1_{VV} \quad S^1_{HH} - S^1_{VV} \quad 2S^1_{HV}]^T$

Image 2: Scattering Matrix: $[S_2] = \begin{bmatrix} S^2_{HH} & S^2_{HV} \\ S^2_{VH} & S^2_{VV} \end{bmatrix}$

Scattering Vector 2: $\vec{k}_2 = \frac{1}{\sqrt{2}}[S^2_{HH} + S^2_{VV} \quad S^2_{HH} - S^2_{VV} \quad 2S^2_{HV}]^T$

Image formation: $i_1 = \vec{w}_1^+ \cdot \vec{k}_1$ and $i_2 = \vec{w}_2^+ \cdot \vec{k}_2$ … projection of the scattering vector on a (complex) unitary vector $\vec{w}_i$

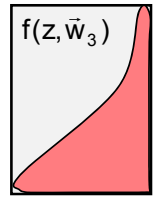
$\vec{w}_i$ used to select a given polarisation out of all possible polarisations provided by [S]

Example: $S_{HH} + S_{VV}$ image: $\vec{w} = [1 \quad 0 \quad 0]^T \rightarrow i = \vec{w}^+ \cdot \vec{k}_j = \frac{1}{\sqrt{2}}(S^j_{HH} + S^j_{VV})$

S_{HH} image: $\vec{w} = [1/\sqrt{2} \quad 1/\sqrt{2} \quad 0]^T \rightarrow i_j = \vec{w}^+ \cdot \vec{k}_j = S^j_{HH}$

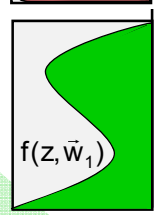
Polarisation 3 ($\underline{w}_3$):

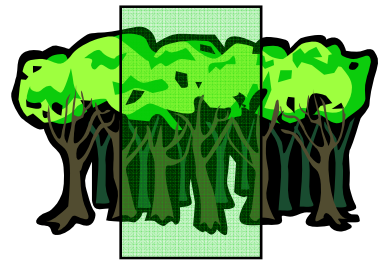
$$\tilde{\gamma}_{Vol}(f(z,\vec{w}_3)) = e^{ik_z z_o} \frac{\int_o^{h_v} f(z,\vec{w}_3)\, e^{ik_z z} dz}{\int_o^{h_v} f(z,\vec{w}_3)\, dz}$$



Polarisation 1 ($\underline{w}_1$):

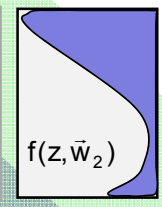
$$\tilde{\gamma}_{Vol}(f(z,\vec{w}_1)) = e^{ik_z z_o} \frac{\int_o^{h_v} f(z,\vec{w}_1)\, e^{ik_z z} dz}{\int_o^{h_v} f(z,\vec{w}_1)\, dz}$$





Polarisation 2 ($\underline{w}_2$):

$$\tilde{\gamma}_{Vol}(f(z,\vec{w}_2)) = e^{ik_z z_o} \frac{\int_o^{h_v} f(z,\vec{w}_2)\, e^{ik_z z} dz}{\int_o^{h_v} f(z,\vec{w}_2)\, dz}$$



$f(z,\vec{w})$ …vertical reflectivity function

Polarimetric SAR Interferometry

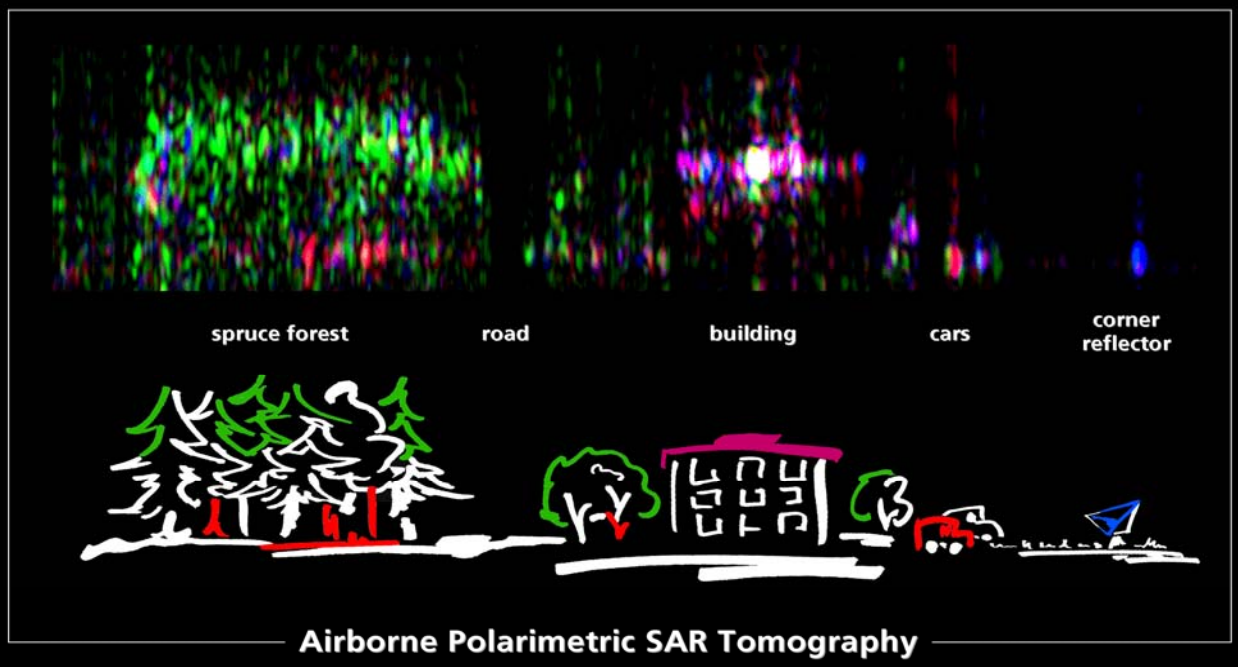
By changing the polarisation the contrast between the individual components consisting the vertical reflectivity f(z) of a (volume) scatterer changes.

Interferometric Coherence: Volume Decorrelation

Amplitude Image HH | Sp. Baseline 16m | Opt 1 | HH | Opt 3

DLR

0 — 1

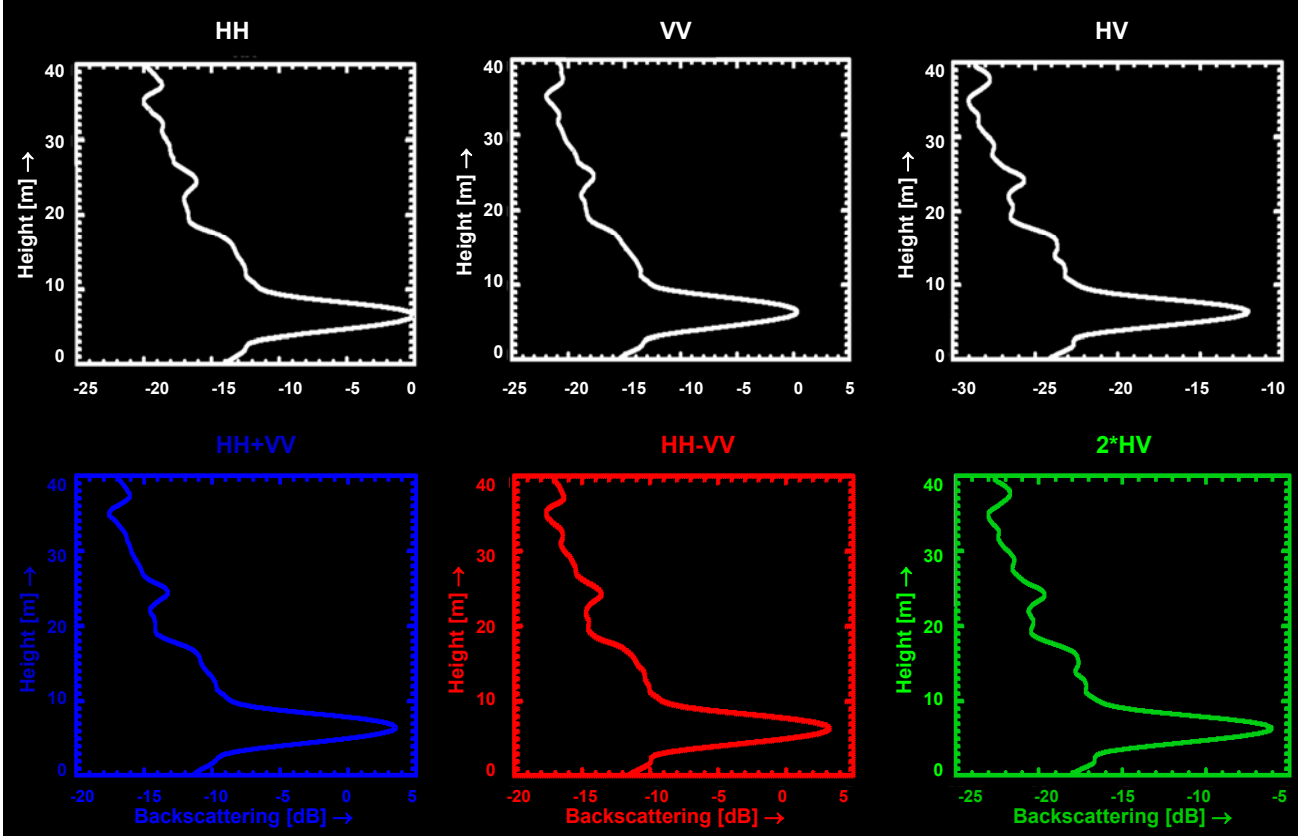


Airborne Polarimetric SAR Tomography

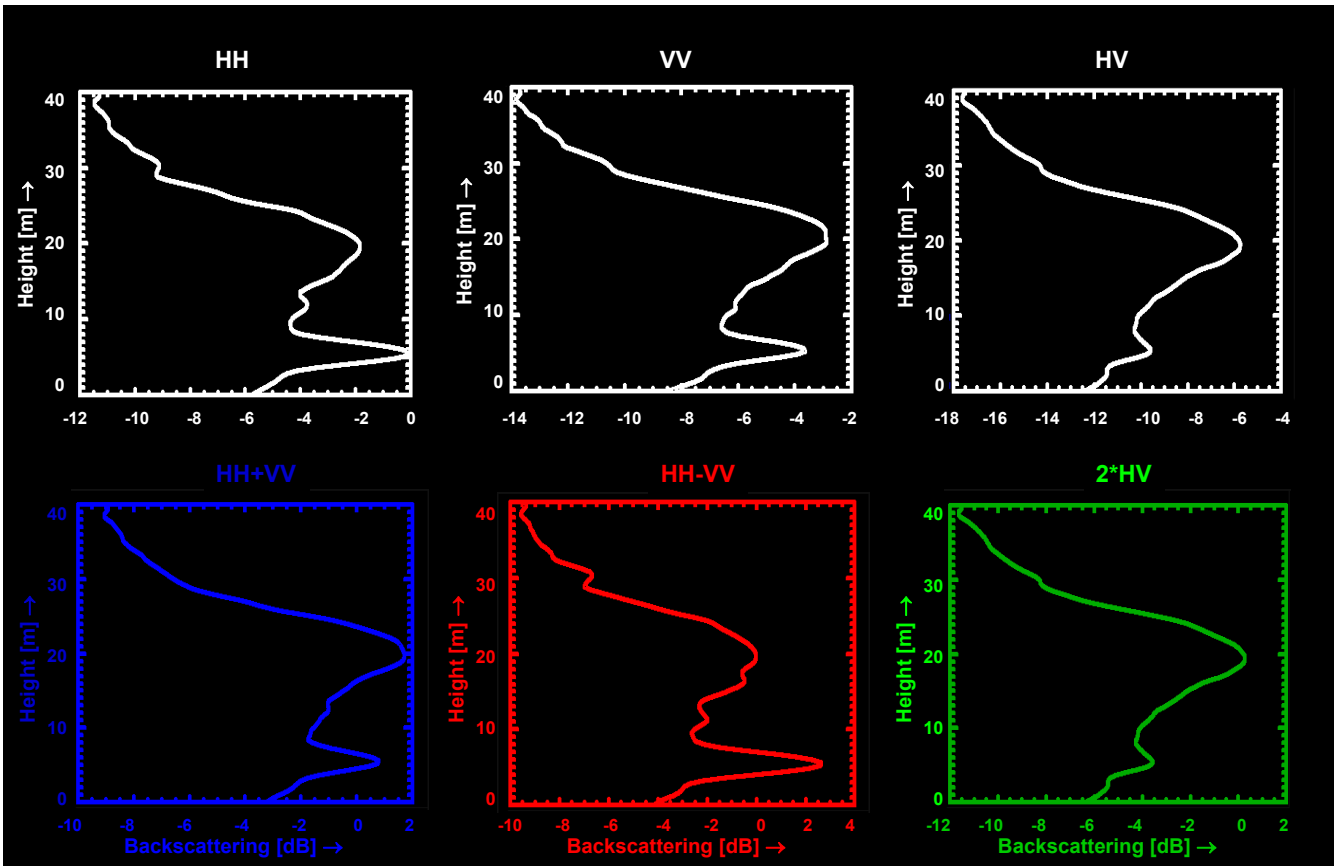
Upper image: Polarimetric color composite (L-band) of a tomographic slice in the height/azimuth-direction
HH+VV, HH-VV, 2*HV

Lower image: Schematic view of the imaged area

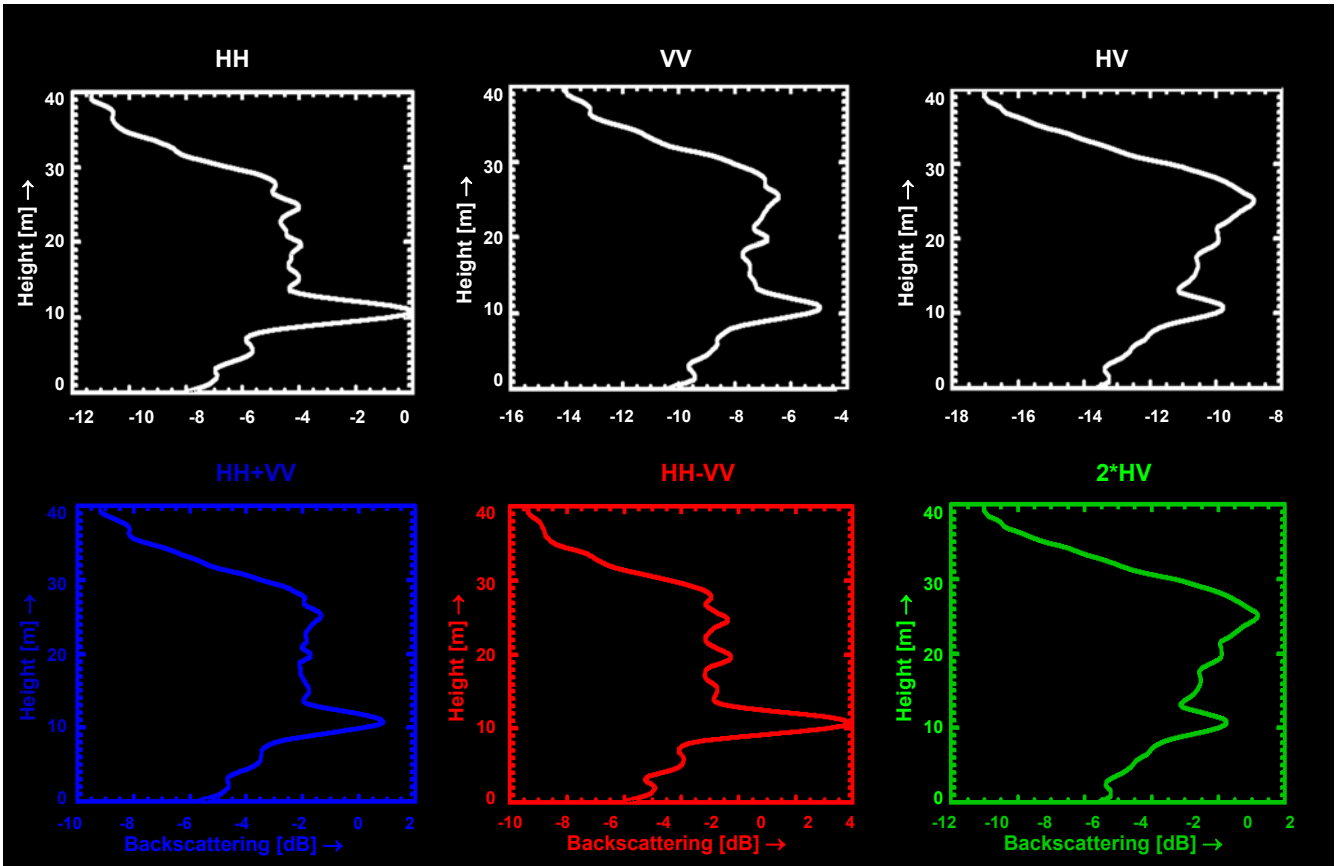
Reigber, A. et al: "First Demonstration of Airborne SAR Tomography using Multibaseline L-Band Data". IEEE Trans. on Geosc. and Remote Sensing, Vol. 38 (5), Sept. 2000



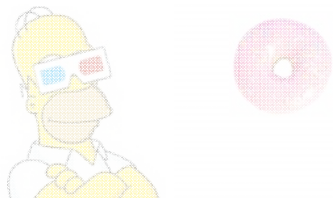
Bare Surface Backscattering Profiles



Spruce Forest Backscattering Profiles (15-20 m height)

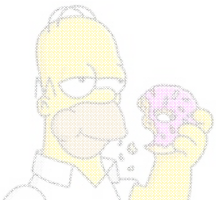


Mixed Forest Backscattering Profiles (12-20 m height)

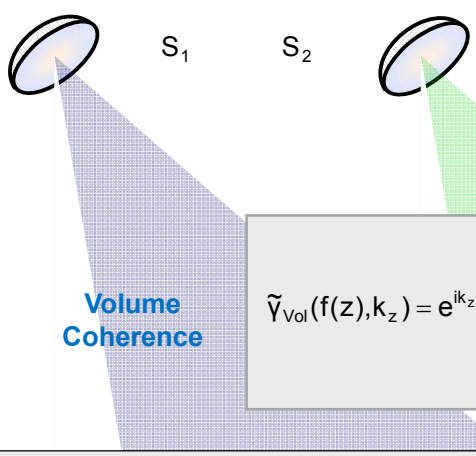


▶ How to set up an observation space for SAR Tomography

▶ How to extract 3-D information content from SAR Tomographic acquisitions



▶ How to use SAR Tomography in the applications

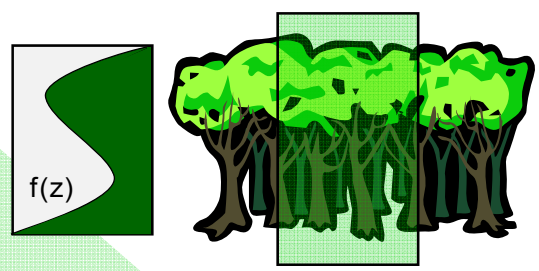


Interferometric Coherence

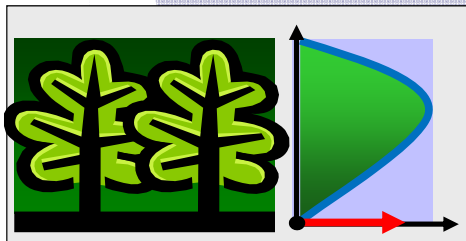
$$\tilde{\gamma}(S_1\ S_2) = \frac{< S_1\ S_2^* >}{\sqrt{< S_1\ S_1^* > < S_2\ S_2^* >}}$$

Volume Coherence

$$\tilde{\gamma}_{Vol}(f(z),k_z) = e^{ik_z z_0} \frac{\int_0^{h_v} f(z)\ e^{ik_z z} dz}{\int_0^{h_v} f(z)\ dz}$$



2 Layer Inversion Model



Volume Layer Ground Layer

$$f(z) = m_V f_V(z) + m_G \delta(z - z_0)$$

$$\tilde{\gamma}_{Vol}(\vec{w}) = \exp(i\varphi_0)\frac{\tilde{\gamma}_V + m(\vec{w})}{1 + m(\vec{w})}$$

$f_V(z)$ … volume reflectivity function

Volume Coherence

$$\tilde{\gamma}_V = \frac{I}{I_0}$$

$$I = \int_0^{h_V} \exp(i\kappa_z z')f_V(z')dz' \qquad I_0 = \int_0^{h_V} f_V(z')dz'$$

$$m(\vec{w}) = \frac{m_G(\vec{w})}{m_V(\vec{w})I_0} \qquad \kappa_z = \frac{\kappa\Delta\theta}{\sin(\theta_0)}$$

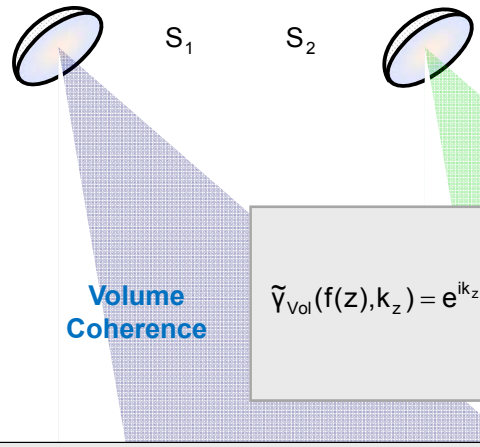
$f_V(z)$ parameterised with N param

Volume Height h_V

Topography φ_0

G/V Ratio $m(\vec{w})$

3+N Unknowns

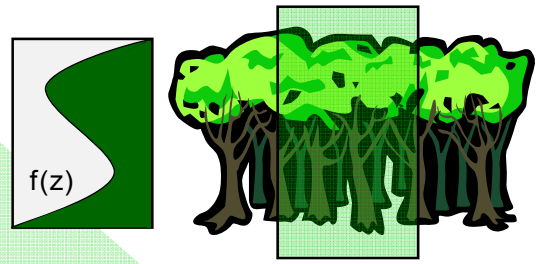


Interferometric Coherence

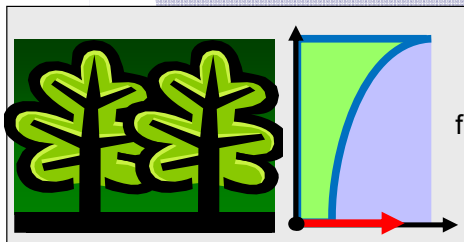
$$\tilde{\gamma}(S_1\ S_2) = \frac{\langle S_1\ S_2^* \rangle}{\sqrt{\langle S_1\ S_1^* \rangle \langle S_2\ S_2^* \rangle}}$$

Volume Coherence

$$\tilde{\gamma}_{Vol}(f(z),k_z) = e^{ik_z z_0} \frac{\int_0^{h_v} f(z)\, e^{ik_z z} dz}{\int_0^{h_v} f(z)\, dz}$$



2 Layer Inversion Model



Volume Layer / Ground Layer

$$f(z) = m'_V e^{\left(\frac{2\,\sigma\,z}{\cos\theta_0}\right)} + m'_G \delta(z - z_0)$$

$$f_V(z) = e^{\left(\frac{2\,\sigma\,z}{\cos\theta_0}\right)}$$ … volume reflectivity function = exponential function

$$\tilde{\gamma}_{Vol}(\vec{w}) = \exp(i\varphi_0) \frac{\tilde{\gamma}_V + m(\vec{w})}{1 + m(\vec{w})}$$

Volume Coherence

$$\tilde{\gamma}_V = \frac{I}{I_0}$$

$$I = \int_0^{h_V} \exp(i\kappa_z z')\, m_V \exp\left(\frac{2\,\sigma\,z'}{\cos\theta_0}\right) dz'$$

$$I_0 = \int_0^{h_V} m_V \exp\left(\frac{2\,\sigma\,z'}{\cos\theta_0}\right) dz'$$

$$m(\vec{w}) = \frac{m_G(\vec{w})}{m_V(\vec{w}) I_0}$$

$$\kappa_z = \frac{\kappa \Delta\theta}{\sin(\theta_0)}$$

$\sigma(z) = \sigma$ "Volume Extinction"

Volume Height h_V

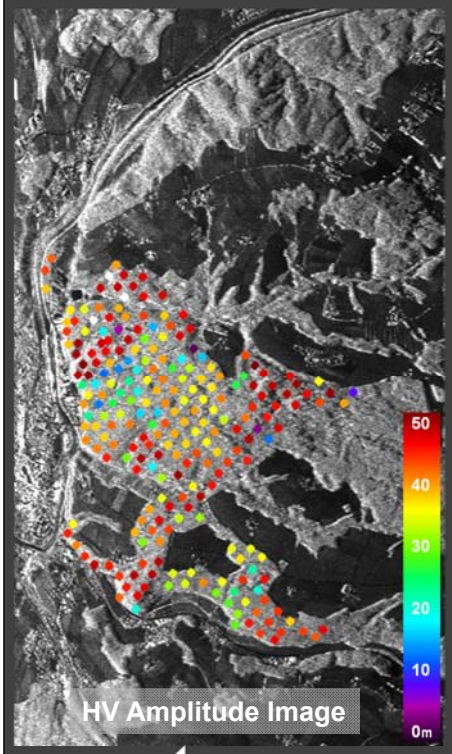
Topography φ_0

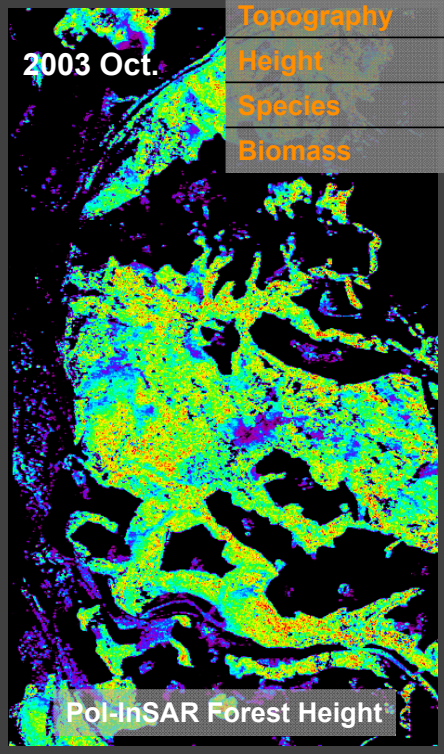
G/V Ratio $m(\vec{w})$

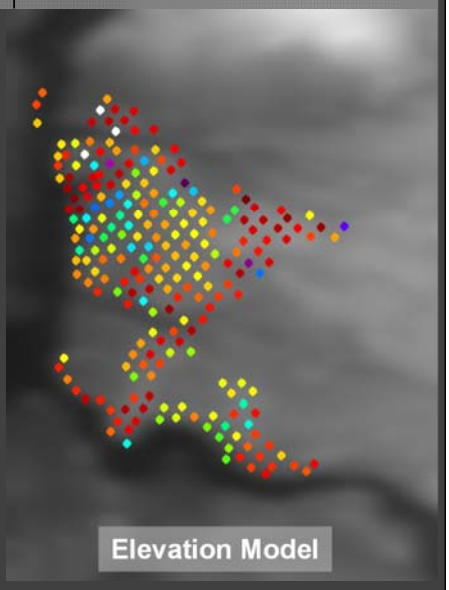
4 Unknowns

Traunstein Test Site

Forest type	Temperate
Topography	Moderate slopes
Height	25 ~ 35m
Species	N. Spruce, E. Beech, White Fir
Biomass	40 ~ 450 t/ha

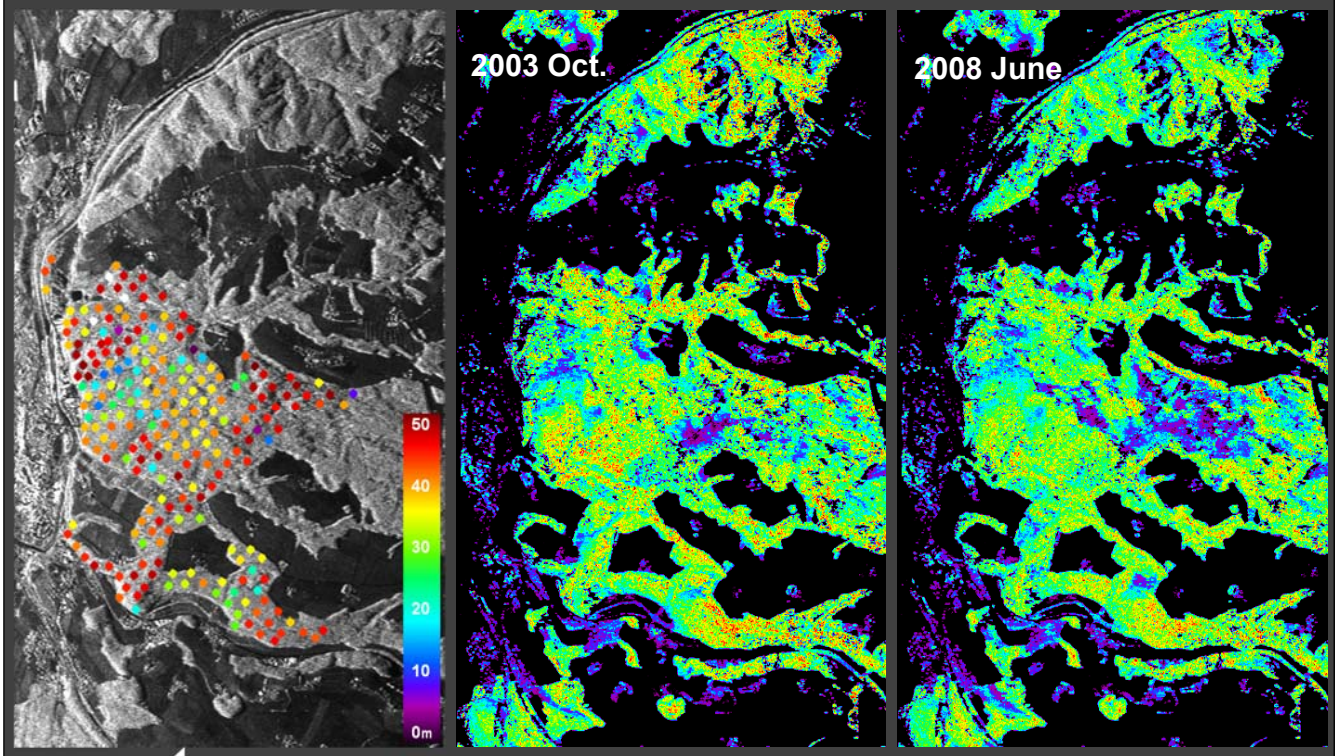




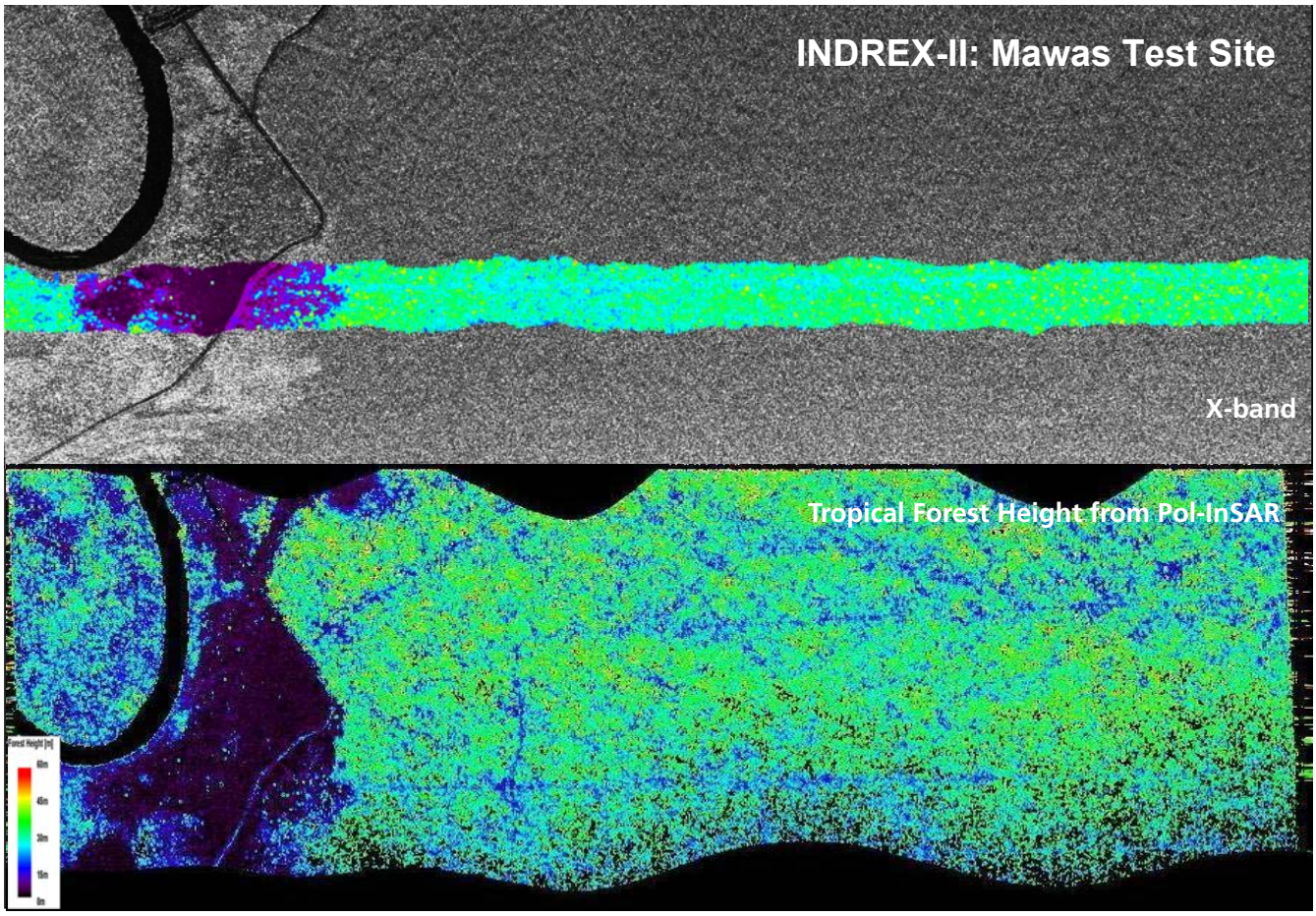


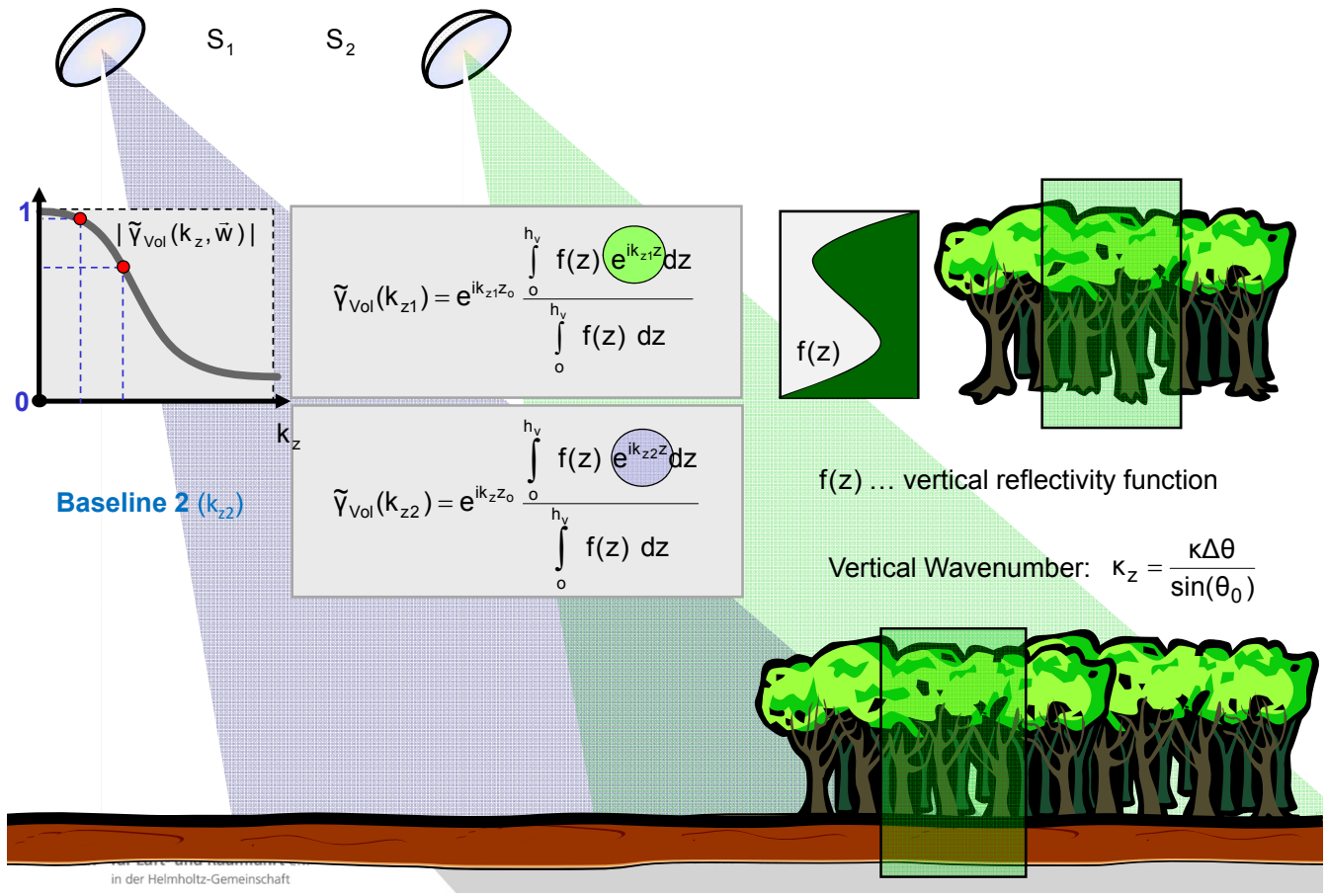
DLR

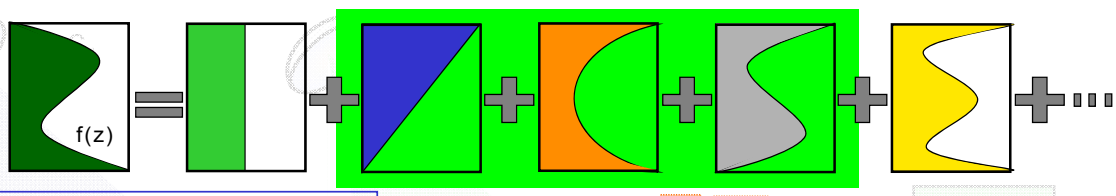
Traunstein Test Site



DLR







$$\tilde{\gamma}_{Vol}(f(z)) = e^{ik_z z_o} \frac{\int_o^{h_v} f(z)\, e^{ik_z z} dz}{\int_o^{h_v} f(z)\, dz}$$

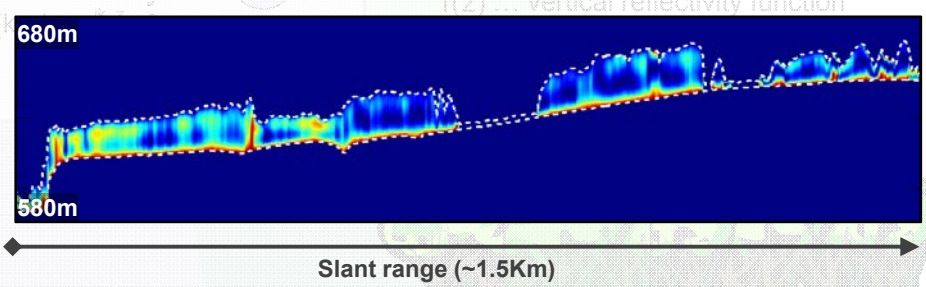
$$\int_0^{h_v} f(z)\, e^{ik_z z} dz = \frac{h_v}{2} e^{i\frac{k_z h_v}{2}} \int_{-1}^{1} (1 + f(z'))\, e^{i\frac{k_z h_v}{2} z'} dz'$$

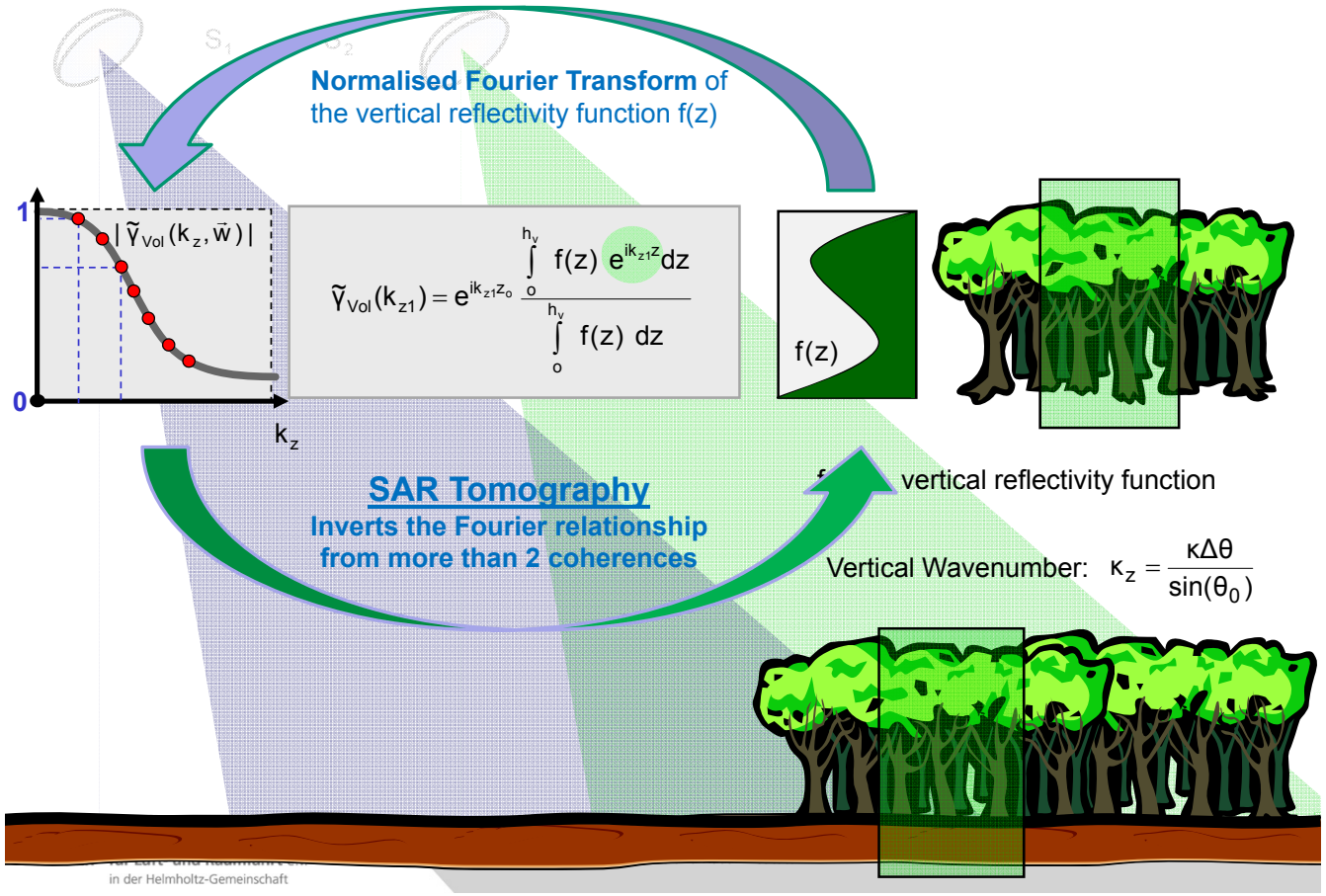
$$\int_0^{h_v} f(z)\, dz = \frac{h_v}{2} \int_{-1}^{1} (1 + f(z'))\, dz'$$

Fourier Legendre Series: $f(z') = \sum_n a_n P_n(z')$ where $a_n = \frac{2n+1}{2} \int_{-1}^{1} f(z') P_n(z') dz'$

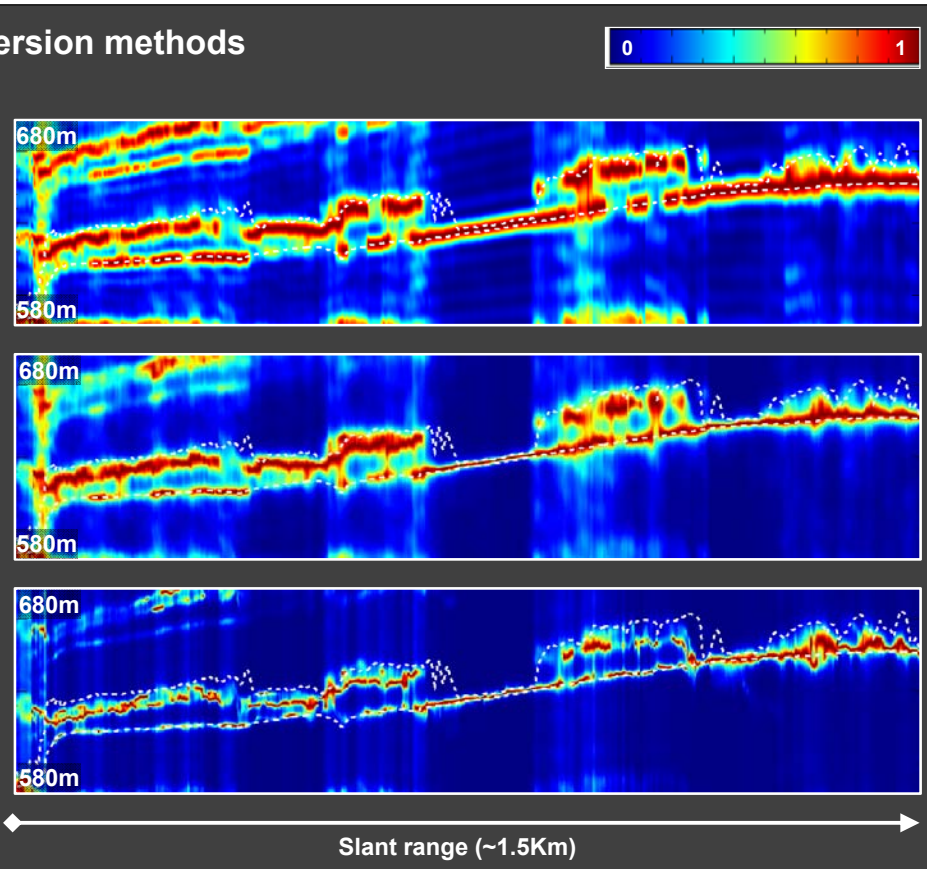
Traunstein forest, L-band

Dual-baseline coherence tomography





SAR Tomography: inversion methods



Direct Fourier inversion

Adaptive spectral Estimation (Capon)

Height dependent imaging

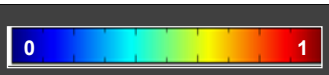
Model-based inversion

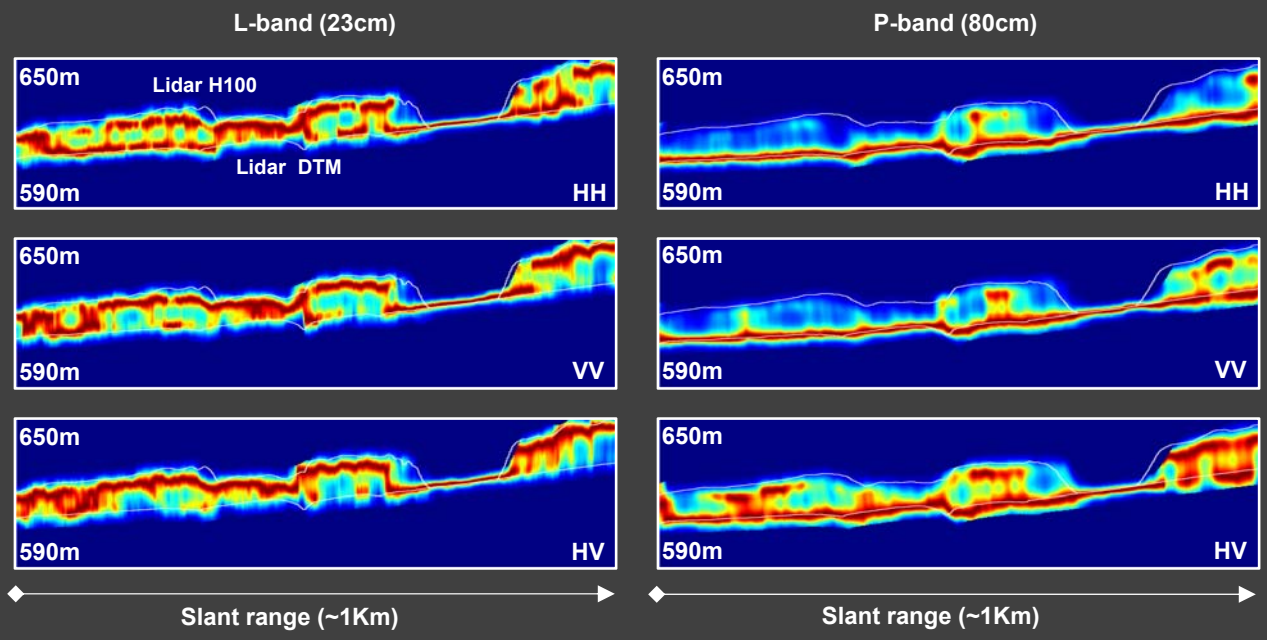
Extraction of the height locations of the vegation layers

DLR

Traunstein site (Germany), 5 tracks (5, 10,15, 25m) – L-band

Dependence on frequency: L- vs P-band

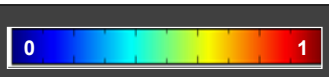


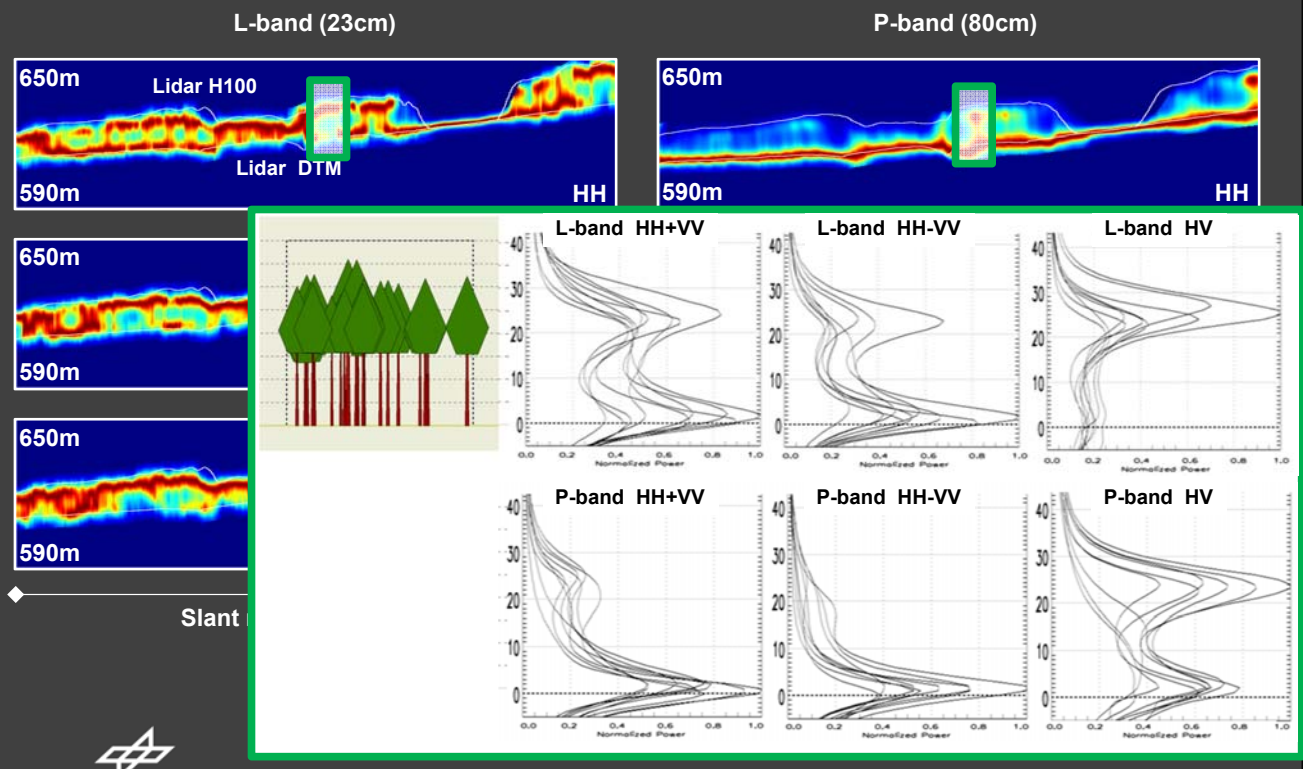


DLR

Traunstein site (Germany), 4 tracks L / 5 tracks P, E-SAR, Capon

Dependence on frequency: L- vs P-band

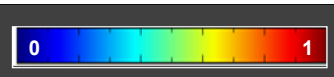




DLR

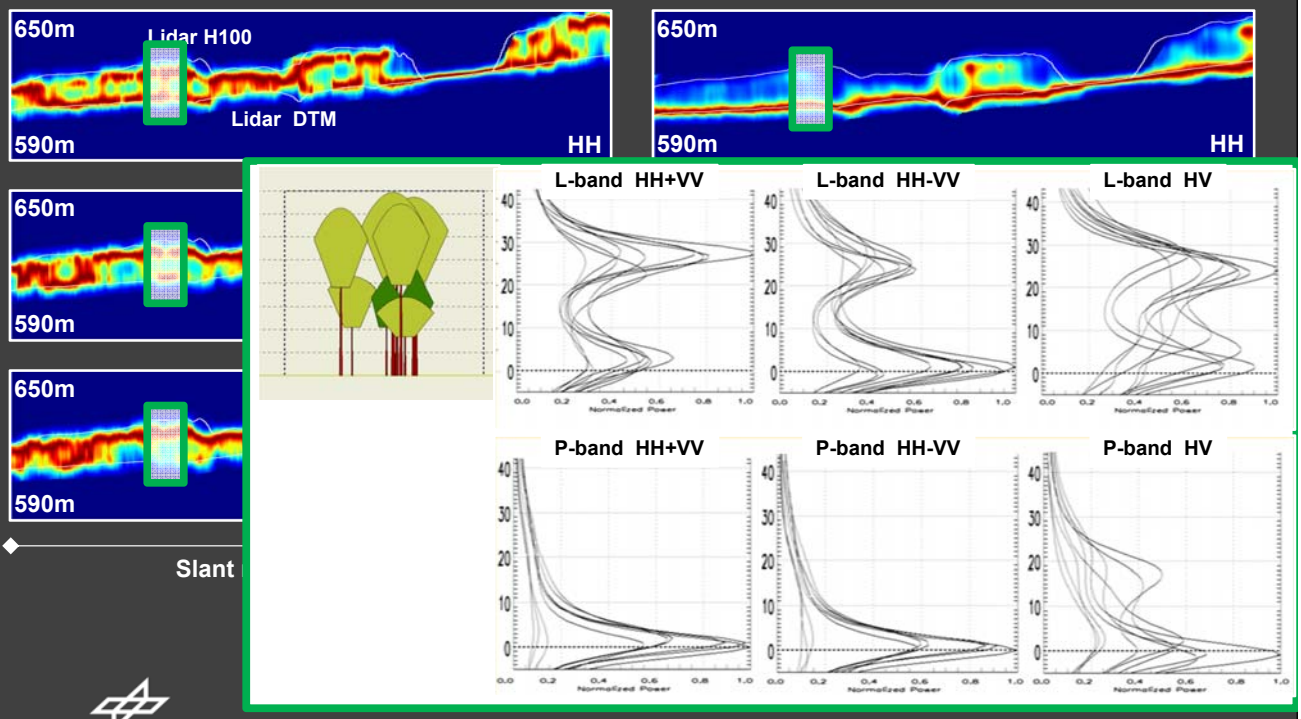
Traunstein site (Germany), 4 tracks L / 5 tracks P, E-SAR, Capon

Dependence on frequency: L- vs P-band



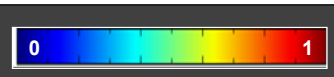
L-band (23cm)

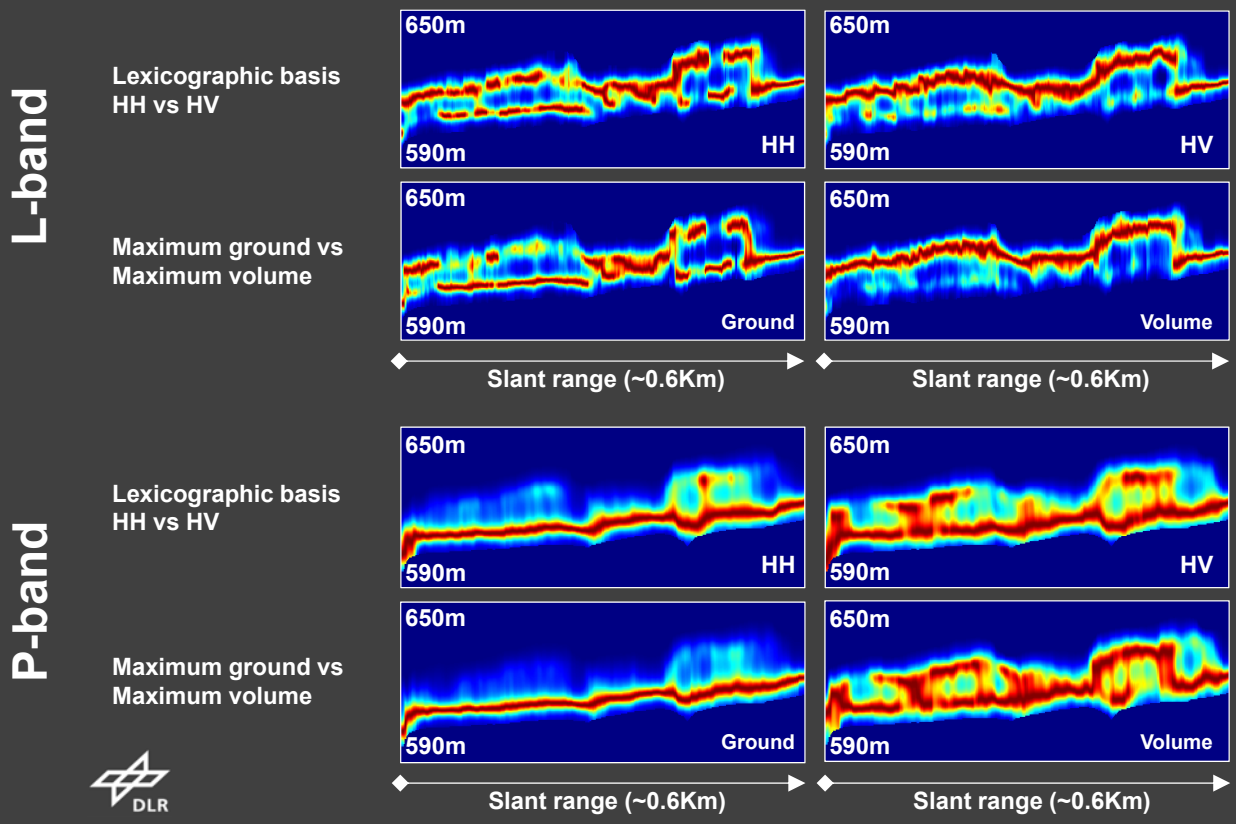
P-band (80cm)



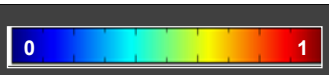
Traunstein site (Germany), 4 tracks L / 5 tracks P, E-SAR, Capon

Dependence on polarization





Dependence on moisture and seasons



Dry day
10 Jun. 2008

Hor. bas.: 0, 5, 10, 15, 25

Wet day
12 Jun. 2008

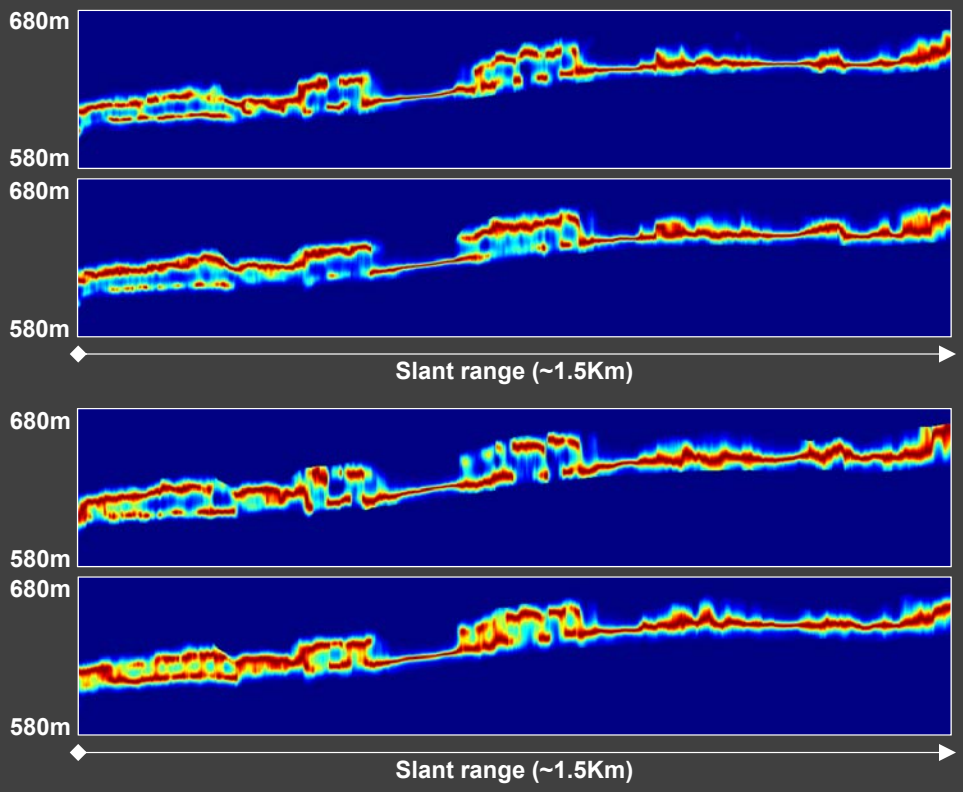
Hor. bas.: 0, 5, 10, 15, 25

Spring
11 May 2009

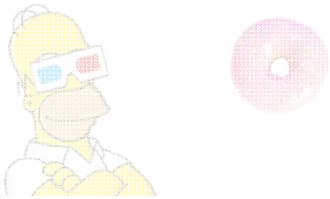
Hor. bas.: 0, 5, 10, 15

Autumn
28 Oct. 2009

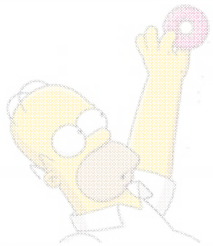
Hor. bas.: 0, 5, 10, 15



DLR



► How to set up an observation space for SAR Tomography

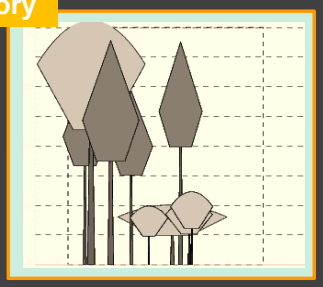


► How to extract 3-D information content from SAR Tomographic acquisitions

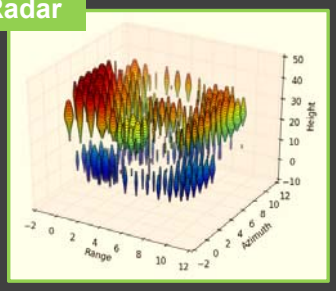
► How to use SAR Tomography in the applications

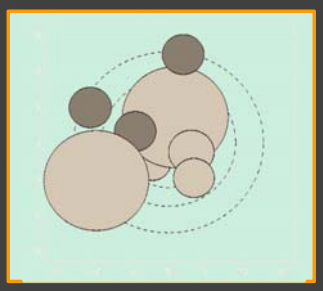
3D Forest Reflectivity

Inventory

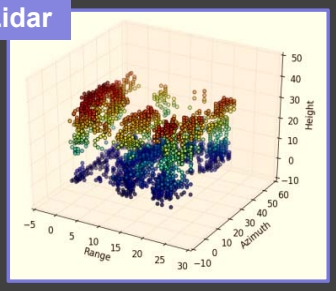


Radar



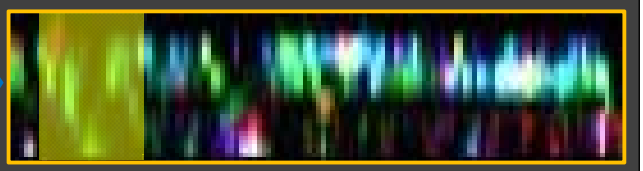


Lidar



Using multiple Pol-InSAR acquisitions
SAR Tomography
allows the estimation of 3-D Radar reflectivity
of natural volume scatterers.

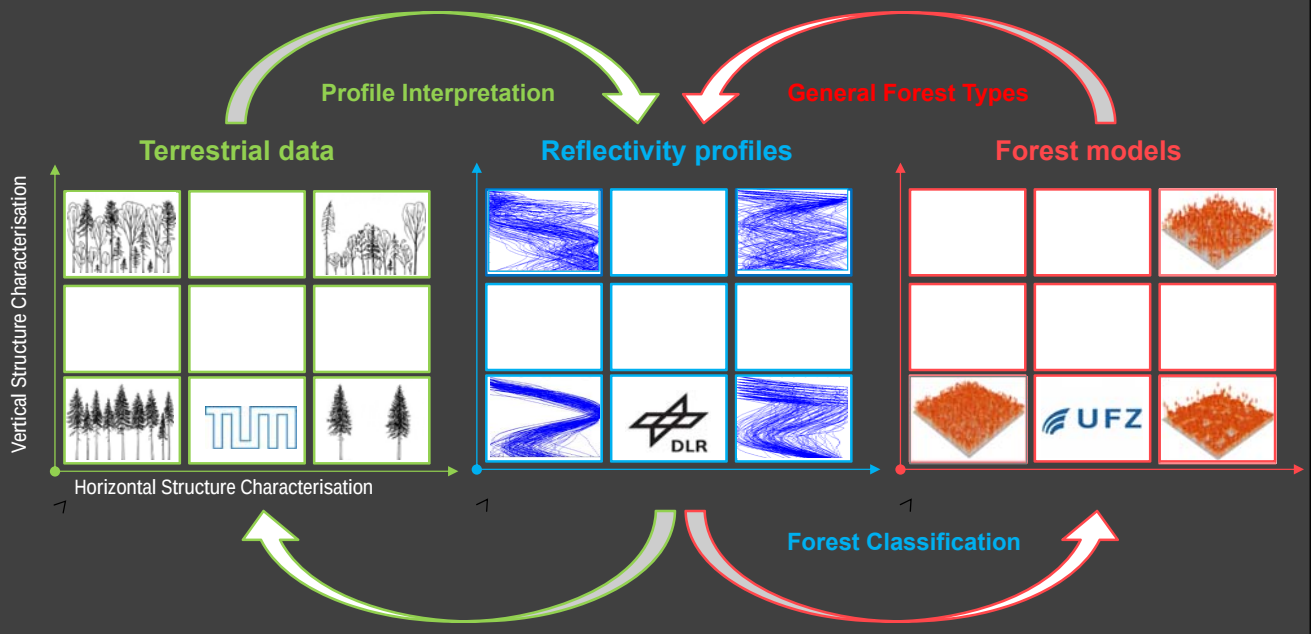
Forest Transect



Radar Reflectivity Transect

HELMHOLTZ
ASSOCIATION

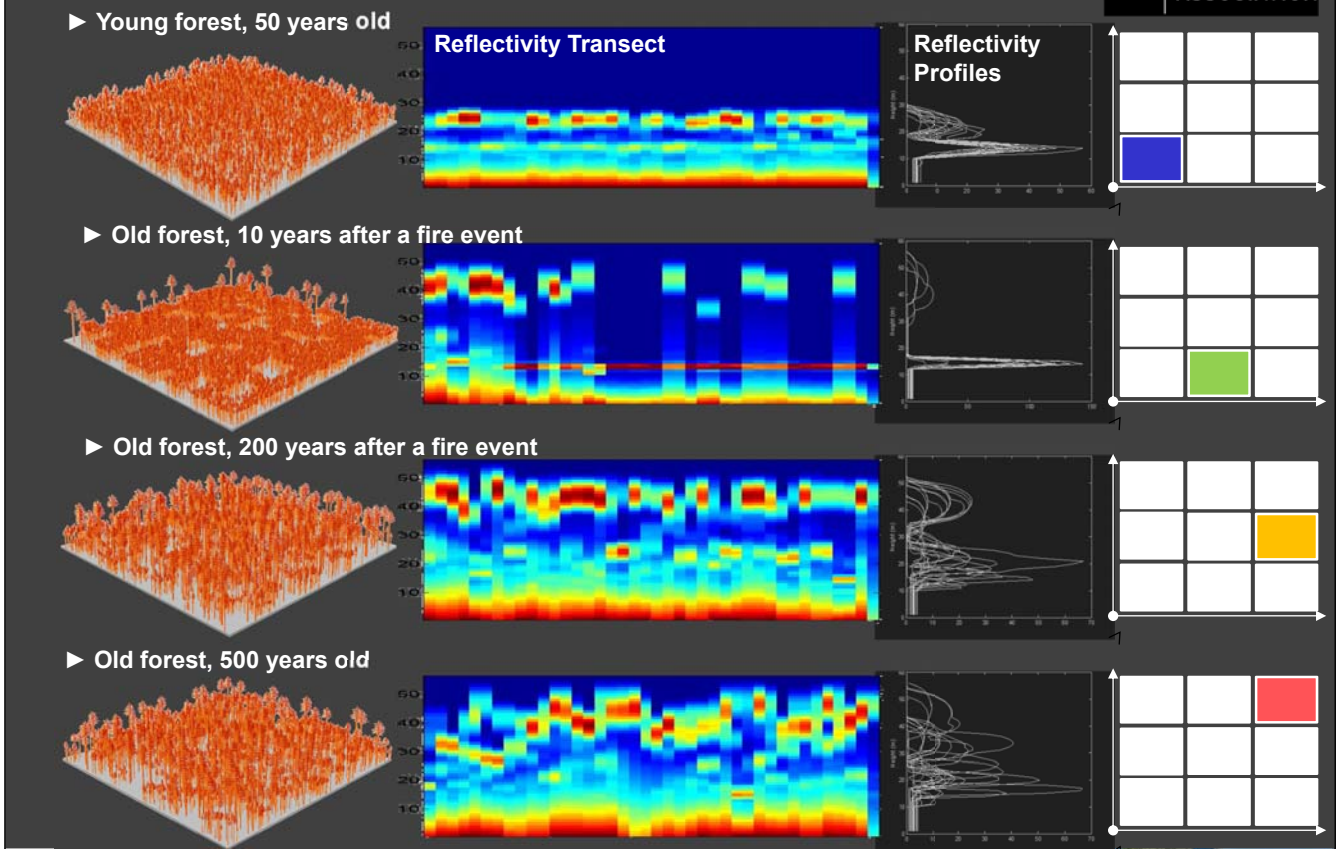
Forest Structure Characterisation



HELMHOLTZ
ASSOCIATION

Forest Structure Characterisation

HELMHOLTZ ASSOCIATION

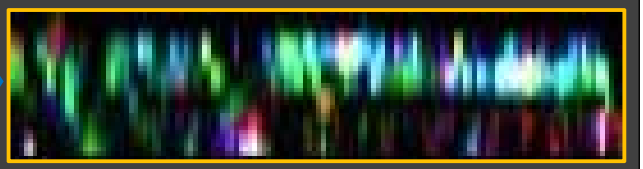


3D Forest Reflectivity

Using multiple Pol-InSAR acquisitions
SAR Tomography
allows the estimation of 3-D Radar reflectivity
of natural volume scatterers.

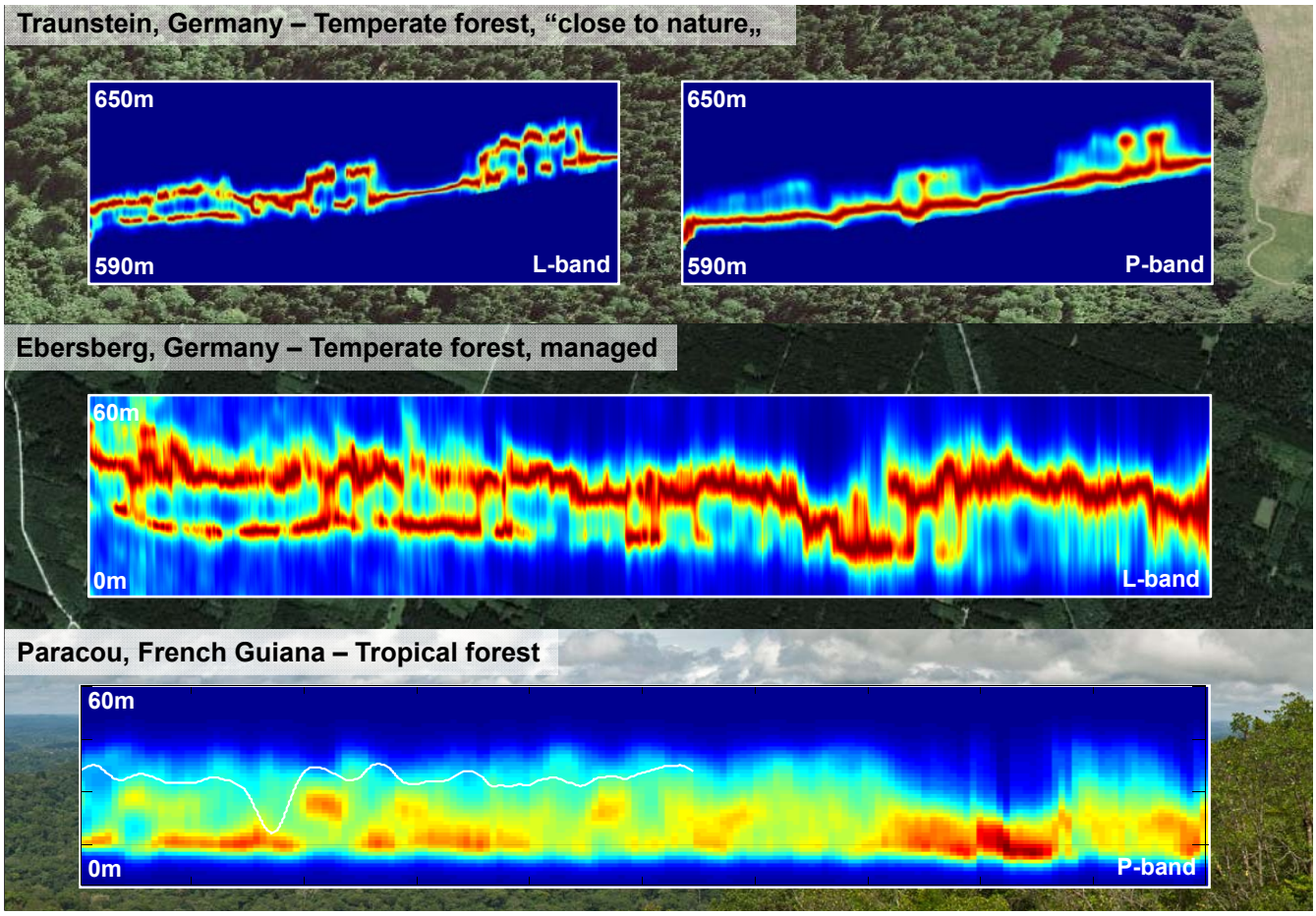
3D Radar Reflectivity Maps at L-band:
- Globally, Wall-to-wall;
- Voxels of 10x10x10m^3;
- Seasonal / Bi-annual.

Forest Transect



Radar Reflectivity Transect

HELMHOLTZ ASSOCIATION



Fundamentals of Synthetic Aperture Radar Tomography

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